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A Multi-scale Sub-Aperture interferometry scheme for Estimating Target Velocity with SAR

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ABSTRACT

A moving target will cause changes in the chirp signal coefficients of signals received by synthetic aperture radar (SAR). By computing these coefficients, we can find the speed of a moving target speed. This paper describes a new approach for estimating the doppler coefficients. The approach uses the sub-aperture interferometry scheme to estimate the chirp signal coefficients. A closed-form expression that describes the relationship between the phase differences and the chirp signal parameters is also derived. It is well known that the radar interferometry can provide the phase differences, but contains the inherent noise. Multi-looking process is one way to reduce the noise deviations, but the measurable spans will become smaller. This can be improved by adopting multi-scale sub-aperture interferometry scheme, as proposed in this paper. Unwrapping the phase differences, we will thus be able to recover the chirp signal coefficients from alias estimates. The maximum measurable span of the coefficients will be significantly larger. Numerical illustrations of the effectiveness of our method are provided.

? . Introduction

By using match filters, the SAR process the chirp signal producing an accurate, high resolution images. The presence of moving targets induces unwanted phase variations, range migration and image degradations. In other words, images that are smeared and ill-positioned with respect to the stationary background are caused. Hence, it is necessary to estimate the relationship between a moving target and the antenna, to improve the SAR imaging (Patrick, 1988; Soumekh, 1994). These estimates also allow us to determine a moving target's velocity, which is the purpose of this paper.

There has been much work on how to estimate the phase coefficients. Based on the fact that a moving target and its stationary background will induce different doppler spectra, some detection methods were proposed (Raney, 1971; Freeman 1987). These methods require the use of a high pulse repetition frequency *(prf)*. They perform poorly, as moving targets have only small velocity components in range direction. Werness et al.(1990) proposed an algorithm that can produce a fine resolution SAR image of moving targets by assuming multiple prominent points, which can be separated and have no phase interference with each other. These requirements are generally not met when range migration occurs or the spatial resolution is not fine enough. Chen et al.(1992) and Soumekh(1994) have described the relationship between the phase coefficients and the center frequency of doppler spectra based on the short time Fourier transform(STFT). It is well known that the STFT resolution is limited, in both the time and in the frequency domain. Furthermore this method suffers from smearing and side-lobe leakage. Other methods using maximum likelihood estimation perform well at low SNR(Besson, 1999; Peleg, 1991; Barbarossa, 1992), but they have a highly computational complexity.

We can obtain the phase differences by the interferometry operation,. Then we can derive the signal coefficients from these phase differences(kuo, 2000). However, the interferometry operation causes the noise deviations to become larger, which leads the estimation to fail. Lee et al.(1994) have proved that multi-look processing can improve the phase accuracy, and we have determined that the sub-aperture interferometry scheme can decrease the noise deviation.

This paper describes an estimation algorithm to find the chirp signal coefficients, based on sub-aperture interferometry scheme. Basically, the phase of the observed signal sequence may be modeled as a polynomial signal embedded in complex Gaussian noise. Since the received signals are wrapped by 2π, which results in aliases for the estimates when the phase differences are larger than 2π, this can come from the sub-apertures sizes, the moving target speed and the SAR system (i.e. wave length, sample spacing and SAR velocity). Thus, the speed of moving targets can only be estimated within some range, hence the measurable span becomes smaller, the reduction of which comes from the sub-aperture size being used. The above dilemma is easily resolved by using a multi-scales sub-apertures interferometry scheme, followed by unwrapping the difference of wrapped phase. The use of this method can effectively reduce the effect of the sub-aperture sizes. We were able to recover the chirp signal coefficients from the alias estimates. Therefore, the maximum measurable span of the coefficients were larger than when only the interferometry operation is used.

? . Joint parameter estimation using multi-scale sub-aperture interferometry

2.1. Multi-scale sub-aperture interferometry

A stationary target area is assumed, for a broadside SAR geometry. During data acquisition, a dynamic target with a constant velocity is assumed to move with respect to a stationary background. We denote the target's velocity vector as (v_x, v_y), which represents the components of the velocity in the directions of the range and the azimuth respectively. We denote the speed along the direction of the radar track to be U. Then the relation between those variables can be expressed as:

$$(v_x, v_y)=(aU,\ bU) \tag{1}$$

where *(a,* b) are ratios between two vectors, normally, |a| and |b| <<1. As mentioned above, we know that *(a*,b) is nearly constant during the integration time of the azimuth compression processing. The relationship between an antenna and a moving target can be expressed as a function of distance(kuo,2000):

$$\begin{aligned} g(Y) &= A(y)\exp\left\{-j\left[\frac{4p\left|\vec{R}\right|}{l}+\frac{4paY}{l}+\frac{2p(1-b)^2Y^2}{l\left|\vec{R}\right|}\right]\right\}+n(Y) \\ &= A(y)\exp\left\{-j\left[f_0+f_1Y+f_2Y^2\right]\right\}+n(Y) \\ &= A(y)\exp\left\{y(Y)\right\}+n(Y) \end{aligned} \tag{2}$$

where

$\vec{R}$ is the vector of the distance from target to radar,

A(t) is the amplitude of the signal,

? is the wave length of the antenna.

Y is sample space for the synthetic aperture of SAR,

f_0, f_1, f_2 are its initial phase, initial frequency and frequency rate, respectively,

$y(Y)$ is the phase of the signal,

n(Y) denotes zero mean complex Gaussian noise.

This is the linear FM form of the signal, where A(Y) is a nuisance parameter and will not be estimated. The first part of eq.(2) gives the initial phase value for the target's slant range in the broadside position. The second term is the doppler shift caused by the target's velocity component in the slant range. The third term is the target's and the antenna's velocity component at the azimuth.

For any given sub-aperture length? , the estimates of the parameters of *g(*Y), in theory, are(kuo,2000):

$$f_2 = \frac{\Delta f(N)}{2h^2} \tag{3}$$

$$f_1 = \frac{\Delta\tilde{f}(N)}{h} \tag{4}$$

Where $\Delta f(N)$ are the phase differences from the frequency rate with a sub-aperture size N, $\Delta\tilde{f}(N)$ are the phase differences from the initial frequency with sub-aperture size N. The limits in (kuo,2000) be improved by using multi-scale sub-apertures? $_1$=N_1y and? $_2$=N_2y. We can then resolve the difference relation between N_1 and N_2 by unwrapping the differences.

However, due to the inherent wrap-around effect of the above method, we observe the phase differences located at $\Delta f(N) \in [0,2p]$. The actual values above are $\Delta j(N) = 2k(N)p + \Delta f(N)$ for some integer k. Supposed that the two sub-aperture length? $_1$ and? $_2$ are used, we can then obtain two sets of phase differences (possibly aliased). Hence, there exists k(?)∈ Z such that:

$$\Delta j\,(N_1) = 2k_1 p + \Delta f\,(N_1) \tag{5}$$

$$\Delta \tilde{j}\,(N_1) = 2k_2 p + \Delta \tilde{f}\,(N_1) \tag{6}$$

$$\Delta j\,(N_2) = 2k_3 p + \Delta f\,(N_2) \tag{7}$$

$$\Delta \tilde{j}\,(N_2) = 2k_4 p + \Delta \tilde{f}\,(N_2) \tag{8}$$

From the above equations, we can obtain the coefficients:

$$\hat{f}_2 = \frac{2p\,(k_3 - k_1) + (\Delta j\,(N_2) - \Delta j\,(N_1))}{2(h_2^2 - h_1^2)} \tag{9}$$

$$\hat{f}_1 = \frac{2p\,(k_4 - k_2) + (\Delta \tilde{j}\,(N_2) - \Delta \tilde{j}\,(N_1))}{h_2 - h_1} \tag{10}$$

Where the sub-aperture lengths are given. Then, the phase differences can be obtained using the sub-aperture interferometry scheme. Thus, we only need to solve the differences between k_1 and k_3 or k_2 and k_4. Then, we can obtain the coefficients of the chirp signal and we can adopt phase unwrapping to solve the above equations.

3.2. Phase unwrapping

In section? , we have defined sub-aperture size η=N*y. Phase unwrapping is the process of estimating $\Delta j\,(N)$ from $\Delta f\,(N)$, by estimating the integer k(N). N is the number of sub-aperture samples, which should be sufficiently large for statistic significance (i.e. on an sample average operation); On our algorithm, the meaning of N is different from other method, in which N denotes the position within the sampled signals. Let the difference operator ? is defined as

$$\nabla\{\Delta f\,(N)\} = \Delta f\,(N+1) - \Delta f\,(N) \qquad -p < \nabla\{\Delta f\,(N)\} \le p \tag{11}$$

$$\begin{aligned}\nabla\{\Delta j\,(N)\} &= \Delta j\,(N+1) - \Delta j\,(N) \\ &= 2p[k(N+1) - k(N)] + \Delta f\,(N+1) - \Delta f\,(N)\end{aligned} \tag{12}$$

Because the values of the difference in the interval $-p < \nabla\{\Delta j\,(N)\} \le p$, which implies that $0 = k(N+1) - k(N)$ then:

$$\nabla\{\Delta j\,(N)\} = \nabla\{\Delta f\,(N)\} \tag{13}$$

eq.(13) states that the phase $\Delta j\,(N)$ can be unwrapped by integrating the wrapped differences $\Delta f\,(N)$ (Itoh, 1982; Ghiglia, 1998).

$$\begin{aligned}\Delta j\,(N) &= 2pk(N) + \Delta f\,(N) \\ &= \Delta j\,(N_0) + \sum_{m=N_0}^{N-1} \nabla\{\Delta f\,(m)\}\end{aligned} \tag{14}$$

where

$\Delta j\,(N_0)$ denotes the initial phase

N_0 denote the initial number of sub-aperture samples

There are two factors that will affect the results of the phase unwrapping operation. One is phase aliasing, and the other is noise. Noise can cause a catastrophic failure during phase unwrapping. In our algorithm, we can reduce the noise effect by averaging all available phase differences. Phase aliasing can be produced from a smaller sampling signal set or inappropriate interferometry operations. According to sampling theory, more than two samples of the highest frequency component per period must be obtained. The Nyquist rate defines this minimum sampling rate. Alternatively, sampling at the Nyquist rate is equivalent to constraining the phase change to less than radians per sample (in magnitude) everywhere. In our algorithm, we assume that the SAR signals should satisfy this condition. The factors of the interferometry operation include the signal properties of f_2 and f_1, or the sub-aperture lengths. We summarize the effects of the phase unwrapping performance as follows:
The unwrapping process is to compare $\Delta f(N+1)$ and $\Delta f(N)$ with the threshold . Depending on the value of the difference, k(N) is decremented or incremented. Unwrapping failures occur when $|\Delta f(N+1) - \Delta f(N)| \succ p$, where the absolute value increases with the signal parameter values of f_2 , f_1. While the absolute value increases, less noise can be tolerated for correct unwrapping. Hence, the performance of the estimator depends on the coefficients, the sub-apertures lengths and the signal to noise ratio. The measurable span of coefficients f_2 , f_1 are confined by $|\Delta f(N+1) - \Delta f(N)| \prec p$.

Next, we will test the algorithm. We use ERS SAR as the simulation system to illustration the process. We also assume that the electromagnetic behavior of the moving targets is similar to the one point scatter response. The magnitude of the slant range is 8.4848e+005 meters, the aircraft altitude is 800000 meters, the azimuth sample space is 3.990574 meters, the total sample number is 1068, the platform ground speed is 6.699028e+03m/sec, the platform heading is 192.036610832947^{o}, the radar wavelength is 0.056666 meters, the target heading is 161^{o}. Assuming that the moving target's velocity varies from -150m/sec to 150m/sec, we can find the frequency rate f_2. The estimated results for $\hat{f}_2$ are plotted in Figure 1.(a). Similar to the previous method(kuo,2000), the estimated results agree very well with the true coefficients. The initial frequency f_1 and the estimated results of $\hat{f}_1$ are displayed in Figure 1.(b). By comparing Figure 1, with Figure 2, (sub-aperture interferometry scheme) we find that the maximum measurable span of the coefficients f_1 within the multi-scale sub-aperture interferometry is significantly larger than for the algorithm merely adopts sub-aperture interferometry.

? . CONCLUSION

In this paper, we develop an algorithm that can estimate the coefficients of the chirp signal in SAR. Through the above analysis, we have proven the relationship between the phase difference and the chirp signal coefficients. We then obtain the target velocity. The estimated coefficients can also be used to refine image quality. Finally, we apply the algorithm to estimate the target speed for simulated data from ERS SAR signals. The computers simulation results agree very well with the theoretically-derived performance. We find that the maximum measurable span for this algorithm of the target's velocity component at the azimuth is almost without limit. On the other hand, the maximum measurable span of the target's velocity component at the slant range is restricted by the wavelength of the SAR system, the SAR velocity and the sample spacing.

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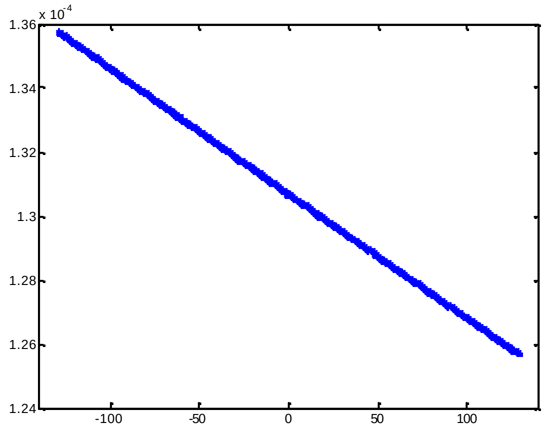
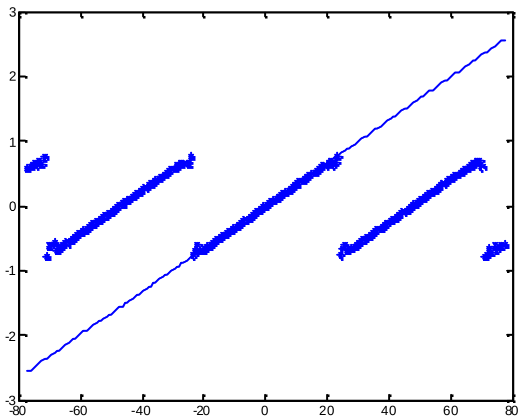


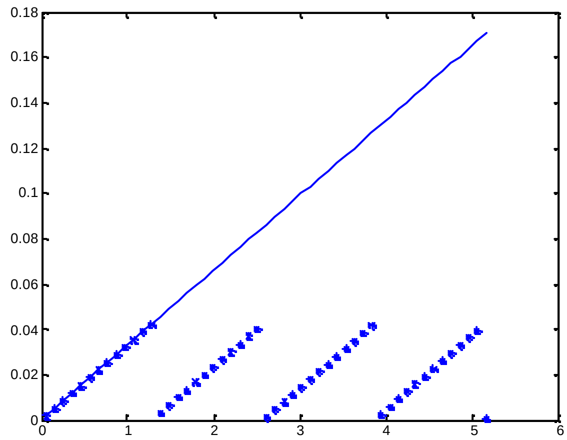
Velocity(m/sec)

Figure1.(a), The frequency rate f_2 and the estimate results $\hat{f}_2$, from the multi-scale interferometry scheme. f_2: solid line, $\hat{f}_2$: "? "



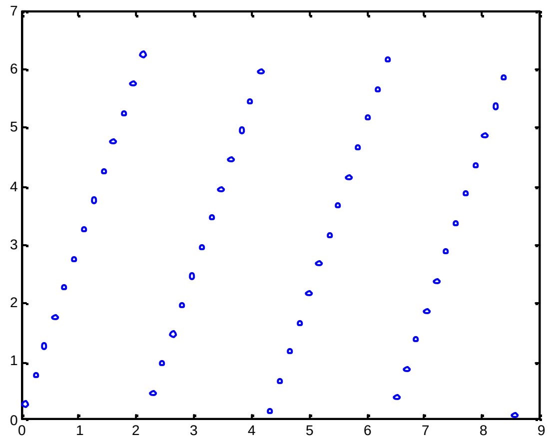
Velocity(m/sec)

Figure1.(b), The initial frequency f_1 and estimate results $\hat{f}_1$, from the multi-scale interferometry scheme. f_1: solid line, $\hat{f}_1$: "? "



Velocity(m/sec)

Figure2.(a), The initial frequency f_1 and estimate results $\hat{f}_1$, from the interferometry scheme. f_1: solid line, $\hat{f}_1$: "? "



Velocity(m/sec)

Figure2.(b), The product $f_1 h$, modulo 2p, from the sub-aperture interferometry scheme.