# A Research on the Method of Simultaneity Least Squares Multi-Point Matching for Producing Digital Terrain Models

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ABSTRACT: The research is concerned with the development of a method for automatically finding a group of corresponding points at the same time using the model of simultaneity least squares multi-point matching with certain constraints. The points are usually selected and distributed in the form of a regular grid in the target image but the distribution of the conjugate points in the search image becomes slightly irregular. The grid points are connected to each other by means of constraints related radiance and geometry. With the aid of the constraints, relationships between the points can be taken into consideration. This matching method is therefore capable of bridging over areas of poor texture with the assistance of knowledge of surrounding points. The idea of the matching method is that least squares matching is simultaneously applied to each of grid points within an area and an affine transformation function is applied for not only minimizing geometric difference between target and search images but also relating between surrounding pixels of adjacent grid point. Due to the relationship, the pixels within the surrounding pixels of different adjacent grid point in the search image are assumed to be capable to find the same corresponding pixel in the target image. It means that these pixels will have the same both position and gray value under different affine transformation parameters for adjacent grid points. Consequently, both the reliability and precision of the matching results can be improved under the constraints of radiance and geometry between grid points. This method has been applied to different areas with different textures and has shown that digital terrain models produced by this method can provide an encouraging accuracy.

# 1. INTRODUCTION

Several different methods and algorithms have been developed for image matching over the past decades. The aim of image matching is to find conjugate on stereo images. Methods have been used in various disciplines but there is no one method that can be used in all disciplines. Generally, they can be divided into area-based matching (ABM) and feature-based matching (FBM). ABM is to find conjugate points on the basis of the gray scale values of pixels around expected points on the stereo images whereas FBM is to determine matched feature from two sets of features detected on the stereo images in advance on the basis of properties of the features. From another point of view, they can also be divided into single point matching and multi-point matching method depending on how many points are matched simultaneously. Matching strategies are developed and applied to improve the efficiency, accuracy or reliability of the matching methods.

The accuracy of image matching by ABM can reach a sub-pixel level, which is good enough for the determination of the heights of matched points. A good approximate point and good texture however are needed to ensure quality of the matching results. Several ways to find the approximate points have been investigated for different situations. One of the ways is to use the results of FBM, which can give higher reliability but less accuracy than ABM, because the pull-in range of FBM is larger than that of ABM and feature points are extracted at places where have high texture. The accuracy of digital terra in models (DTMs) depends not only on the accuracy but also on the distribution of the matched points. The distribution of the matched points can be determined by the sampling pattern chosen. The sampling pattern also affects the efficiency of constructing the DTMs and the degree of automation that can be used to select the points on the images.

DTMs produced using a combination of feature points, grid points and filling back points can give encouraging results (Hsia & Newton, 1999). The feature points located in areas with good image textures were collected automatically by using FBM working with triangular irregular networks (TINs) and an image pyramid. Grid and filling back points with good textures as well as approximation points were generated by means of least squares single point matching (LSSPM). However, height information can be lost as a result of failure of LSSPM because of poor textures. Multi-point matching for generating grid points under the constraints of minimizing the curvature has been developed in order to bridge over areas of poor textures. However, suitable weighting for the constraints is not an easy task since the weights need to be large enough to stabilize the system and still small enough to give suitable effects in areas of poor textures (Rosenholm, 1987).

The aim of this paper is to build a method of simultaneity least squares multi-point matching (SLSMPM) for Producing DTMs. On the basis of the flexibility of the least squares method, the model is able to find simultaneously

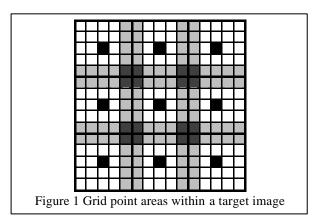
a group of corresponding grid points under not only the constraints of the gray value and position of pixels but also the weighting method of using a robust estimation model. With the constraints, an increase in the reliability of matching results will be made as a result of an increase in the relationships between adjacent points. The influence of land terrains and image textures on the matching results will be reduced.

# 2. METHOD

The model of simultaneity least squares multi-point matching (SLSMPM) is developed for automatically finding a group of corresponding grid points at the same time. The grid points are usually selected and distributed in the form of a regular grid in the target image but the distribution of the conjugate points in the search image becomes slightly irregular. The grid points are connected to each other by means of constraints related radiance and geometry. With the aid of the constraints, relationships between the points can be taken into consideration. This matching method is therefore capable of bridging over areas of poor texture with the assistance of knowledge of surrounding points. The idea of the matching method is that least squares matching is simultaneously applied to each of grid points within an area and an affine transformation function is applied for not only minimizing geometric difference between target and search images but also relating between surrounding pixels of adjacent grid point. It means that these pixels will have the same both position and gray value under different affine transformation parameters for adjacent grid points. Consequently, both the reliability and precision of the matching results can be improved under the constraints of radiance and geometry between grid points. The weight of this model is given using robust estimation method. And corresponding approximated points needed for starting calculation of the model is given by means of a Helmert transformation.

#### 2.1 MATHEMATICAL MODEL

The left image generally is called as a target image and the right one is called as a search image when an image matching process is carried out on stereo images. A target image in this research is cut apart as several square areas of grid points. As shown in figure 1, the whole area of  $15 \times 15$  pixels is cut apart as 9 grid point areas while the size of each point is  $5 \times 5$  pixels. The coordinates of the grid points are the coordinates of black pixels at the center of each grid point area.



Each pixel in a target image produces an equation like Equation 1 according to the grid point area within where the pixel locates. The equation is able to minimize radiometric and geometric differences between a target and a search images in corresponding grid point areas. An affine transformation function, as shown Equation 2, is used to determine the coordinates of the conjugate pixels of any pixels in the target image and a radiometric transformation of two parameters  $(h_{0_i}, h_{1_i})$  is used to adjust the gray scale values of pixels within the *ith* grid point area on the search

image. The positions of the conjugate pixels may not locate as integer pixels. Thus, the bilinear interpolation method is carried out to interpolate their gray scale value from four surrounding pixels according to the distances from the interpolated point to each of the four neighbors.

$$g_t(x_t, y_t) + v_t(x_t, y_t) = h_{0_i} + h_{I_i} g_s(x_{s_i}, y_{s_i}),$$
 (1)  
where  $g_t(x_t, y_t)$  is a gray scale value of a pixel at  $(x_t, y_t)$  coordinates in the target image,  $v_t(x_t, y_t)$  is a residual of a gray scale value of a pixel at  $(x_t, y_t)$  coordinates in the target image,  $g_s(x_{s_i}, y_{s_i})$  is a gray scale value of a pixel at  $(x_{s_i}, y_{s_i})$  coordinates in the search image,

 $h_{0i}$ ,  $h_{1i}$  are a shift and a scale factor of gray scale values in the *ith* grid point,

 $x_t$ ,  $y_t$  are image coordinates of a pixel in the target image,

 $x_{si}$ ,  $y_{si}$  are image coordinates of the conjugate pixel within the *ith* grid point area in the search image,

*i* is the name of grid point.

Each grid point area has its own set of affine transformation parameters. The parameters in Equation 2, for example, belong to the *ith* grid point area. The rotation of the transformation is around the center pixel of the grid point area in the target image. After that, shifts in x and y directions are made in the values of the coordinates of the center pixel of the corresponding grid point area in the search image. Hence, the coordinates of corresponding grid points become the parts of unknown parameters of this model.

$$\begin{cases} x_{s_i} = a_i (x_t - x_{to_i}) + b_i (y_t - y_{to_i}) + x_{so_i} \\ y_{s_i} = c_i (x_t - x_{to_i}) + d_i (y_t - y_{to_i}) + y_{so_i} \end{cases},$$
 (2)

where  $a_i, b_i, c_i, d_i$  are rotation and scale factors for the *ith* grid point area,

 $x_{to_i}$ ,  $y_{to_i}$ ,  $x_{so_i}$ ,  $y_{so_i}$  are image coordinates of the *ith* grid point in the target and search image.

Overlapping pixels come out between two grid point areas as the size of a grid point area in case of Figure 1 is expanded but the center of each grid point is kept at the same location. For example, there are two pixels of overlapping band with slash lines shown in Figure 1 when the size of a grid point area is extended to the size of  $6 \times 6$  pixels. Each pixel in the overlapping area can produce at least two of the Equation 1 but the equations have different sets of affine transformation parameters depending on which grid point area is located in. For example, two equations for a pixel belonging to adjacent areas of the *ith* and (i+1)th grid point can be written as Equation 1 and 3. The affine transformation function of Equation 3 is written as Equation 4. Two another equations in the meantime can be produced for the constraints of radiance and geometry. The radiometric constraint is written as Equation 5 when Equation 3 subtracts from Equation 1 whereas the geometric constraint is written as Equation 6 when Equation 4 subtracts from Equation 2. Solutions to unknown parameters are obtained iteratively, by using the least squares adjustment, until all the corrections to the unknowns are less than a threshold value. Hence, it is necessary to give suit weights and predict good approximate values so that a correct answer can be converged upon quickly.

$$g_t(x_t, y_t) + v_t(x_t, y_t) = h_{0i+1} + h_{1i+1}g_s(x_{si+1}, y_{si+1})$$
(3)

$$\begin{cases} x_{s_{i+1}} = a_{i+1}(x_t - x_{to_{i+1}}) + b_{i+1}(y_t - y_{to_{i+1}}) + x_{so_{i+1}} \\ y_{s_{i+1}} = c_{i+1}(x_t - x_{to_{i+1}}) + d_{i+1}(y_t - y_{to_{i+1}}) + y_{so_{i+1}} \end{cases}$$
(4)

$$h_{0_{i}} + h_{I_{i}} g_{s}(x_{s_{i}}, y_{s_{i}}) - h_{0_{i+1}} - h_{I_{i+1}} g_{s}(x_{s_{i+1}}, y_{s_{i+1}}) = 0$$

$$(5)$$

$$\begin{cases} a_{i}(x_{t}-x_{to_{i}})+b_{i}(y_{t}-y_{to_{i}})+x_{so_{i}}-a_{i+1}(x_{t}-x_{to_{i+1}})-b_{i+1}(y_{t}-y_{to_{i+1}})-x_{so_{i+1}}=0\\ c_{i}(x_{t}-x_{to_{i}})+d_{i}(y_{t}-y_{to_{i}})+y_{so_{i}}-c_{i+1}(x_{t}-x_{to_{i+1}})-d_{i+1}(y_{t}-y_{to_{i+1}})-y_{so_{i+1}}=0 \end{cases}$$

$$(6)$$

## 2.2 WEIGHTING MODEL

The robust estimation method is applied to decrease the influence of pixels with larger residuals on the least squares estimation process. The change weight function p, written in Equation 7, in the robust estimation process is used to change the weight of the observations depending on their residuals in each iteration. In addition, part of weight for observation of Equation 1 is given on the basis of the difference value (i.e. the v value in Equation 7) between the parallax value of a pixel and the parallax mean value of surrounding grid points. The parallax value is the summation of parallax values in x and y directions, whereas the parallax mean value is weighted mean of parallax values of surrounding grid points. The weight is an inverse to the distance in plan from the pixel to the grid point. The  $\sigma_0$  in

Equation 7 at this stage is also determined by the parallax value of surrounding grid points.

$$\begin{cases} 1 & when |v| < 2\sigma_{0}, \\ exp\left[-0.05 \left(\frac{|v|}{\sigma_{0}}\right)^{4.4}\right] & for the first three iterations when |v| > 2\sigma_{0}, \\ exp\left[-0.05 \left(\frac{|v|}{\sigma_{0}}\right)^{3.3}\right] & for the fourth iteration and afterwards when |v| > 2\sigma_{0} \end{cases}$$

$$(7)$$

where v is the residual of an observation and  $\sigma_0$  is the standard deviation of the residuals.

#### 2.3 INITIAL VALUE

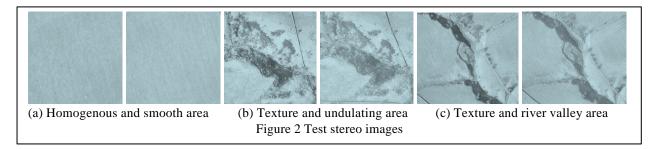
To start the estimation process, initial values of radiometric transformation parameters  $(h_{0_i}, h_{1_i})$  and geometric transformation parameters  $(a_i, b_i, c_i, d_i, x_{so_i}, y_{so_i})$ , have to be given. The initial values of  ${}^h o_i, {}^h l_i, a_i, b_i c_i, d_i$  for each grid point can be (0,1,1,0,0,1). The initial values of  $x_{so_i}$  and  $y_{so_i}$  is the approximate conjugate point positions of grid points in a target image. Therefore, the approximate positions in this research are determined by using Helmert transformation function, as shown in Equation 8, whose four parameters are calculated by two pairs of corresponding points sited around the two opposite corners of a matching area.

$$\begin{cases} x_S = ax_t + by_t + c \\ y_S = bx_t + ay_t + d \end{cases}$$
 (8)

## 3. EXPERIMENTAL RESULTS

#### 3.1 TEST IMAGES

Data used in this research is the parts of an aerial photography, at an approximate scale of 1:12000, taken with a Wild camera fitted with a wide angler lens (f=152.72mm). The base-height ratio (B/H) of the photography was 0.56. The photography was scanned digital images at 500 dpi. Three areas of the digital images are shown in Figure 2. Images shown in Figure 2 (a) ( $177\times124$  pixels), cover an area with a homogenous image and smooth terrain where as images shown in Figure 2(b) ( $285\times185$  pixels) and (c) ( $319\times299$  pixels) cover areas with texture images and more undulating terrain.



## 3.2 EXPERIMENTAL DES IGN

To realize the influence of image size of a grid point area and the number of overlapping pixels on the quality of this matching model, image sizes in  $15 \times 15$ ,  $21 \times 21$ ,  $25 \times 25$  and  $31 \times 31$  pixels and overlapping pixel numbers in 2, 12 and 14 have been applied to the three areas. Contours, shown in Figure 3, 4 and 5, were plotted by means of the Kriging method of interpolation on the basis of the data points produced by the matching method developed in this research.

#### 3.3 RESULTS AND DISCUSSION

The resulting contours shown in Figure 3 generally are similar except (1) of a and b as well as all of c. This indicts that good contours in the area with homogeneous image and smooth terrain can be obtained on the basis of data points matched by this matching method, especially using 31 × 31 pixels of the grid image size. An increase in the number of overlapping pixels leads to an improvement of the resulting contours as small image size of grid point areas is used. Disagreement of contours, as shown in Figure (1) of a, b and c, took place around the corners where there are no matched points for determining Helmert transformation parameters, mentioned in section 2.3, shows that the disagreement is the results of poor approximate conjugate point position however it can be alleviated by increasing the number of overlapping pixels. The improvement can be made by the constraints of radiance and geometry.

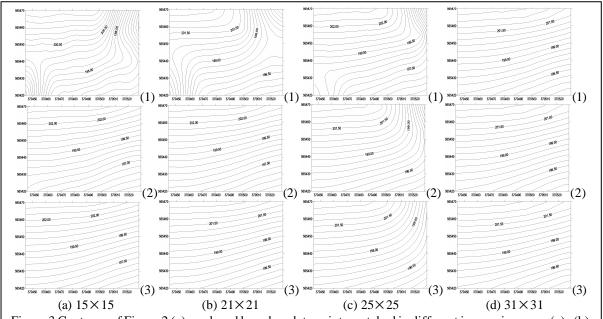
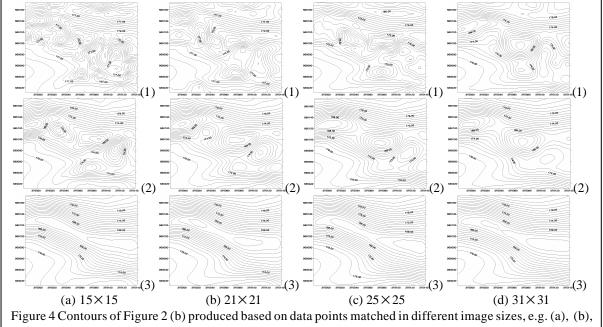
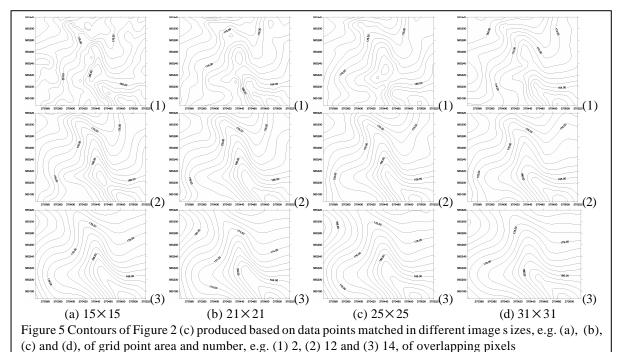


Figure 3 Contours of Figure 2 (a) produced based on data points matched in different image sizes, e.g. (a), (b), (c) and (d), of grid point area and number, e.g. (1) 2, (2) 12 and (3) 14, of overlapping pixels



(c) and (d), of grid point area and number, e.g. (1) 2, (2) 12 and (3) 14, of overlapping pixels

Contours, shown in Figure 4 and 5, display better results when the number of overlapping pixels increases for different image sizes of grid point areas. The influence of the image size on the contours seems very small. However, it can be shown that there is little smoothing effect when the image size is getting larger. This indicates that using a small image size of grid point area gives better resulting contours when the terrain of areas becomes undulating.



According to the distribution of data points, points produced by the matching method can be distributed as uniform as possible since points located in the areas with poor textures or large height variation can be matched automatically and successfully but not by the least squares single point matching. This leads to a result that the research method can give more reliable results.

# 4. CONCLUSIONS

This research has concentrated on the development of algorithms and programs to automatically generate digital terrain models (DTM) in the areas of poor textures or large height variation using grid points collected by means of an image matching method. The program for simultaneity least squares multi-point matching (SLSMPM) has been applied on three test areas. Based on the test results, the following conclusions may be drawn:

With the aids of the constraints of radiance and geometry as well as the weighting method of using a robust estimation model, an increase in the reliability of matching results is given as a result of an increase in the relationships between adjacent points. The influence of land terrains and image textures on the matching results is reduced. Hence, the contours plotted on the basis of the matching results can give encouraging results though in the areas with poor image texture or large height variation.

A bigger image size of the grid point area is recommend when the matching method is applied on the areas of poor image textures and smooth terrains whereas a smaller image size and a bigger number of overlapping pixels are recommend on the areas of large height variation.

There is little smooth effect on the DTM when the number of overlapping pixels becomes bigger. The effect however has not too big to give good resulting contours.

## 5. ACKNOWLEDGMENT

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