ESTIMATING CAMERA FOCAL LENGTH AND ROTATION FROM A SINGLE VANISHING POINT AND SCALE RATIO

Po-Lun Lai*[1] and Alison Sheets[2]
[1]Post Doctoral Researcher, Mechanical and Aerospace Engineering, The Ohio State University
201 W. 19th Ave., Columbus, Ohio 43210, USA
Email: lai.138@osu.edu

[2]Assistant Professor, Mechanical and Aerospace Engineering, The Ohio State University
201 W. 19th Ave., Columbus, Ohio 43210, USA
E-mail: sheets.203@osu.edu

KEY WORDS: Calibration, Vanishing Point, View Geometry, Scale Ratio

ABSTRACT: In this paper, a new approach is presented for estimating camera focal length and rotations relative to a world plane using only the relative positions of four points. This approach provides more flexibility for camera calibration, especially for the cases in which only limited ground truth data is available. First, we leverage the property of scale ratio for two image points to find the camera focal length. The rotation about the selected world reference frame is then determined by analyzing the geometric relationship among the camera center, principle point, vanishing point and induced vanishing line. Camera parameters estimated using this method could also be used as alternative initial approximation for other rigorous calibration techniques. Experimental results show that the proposed approach generates reasonable approximates for the parameters, especially when the vanishing points are obtained from a larger spatial distribution.

1. INTRODUCTION

Among computer vision and photogrammetric processes, recovering the internal and external camera parameters remains a fundamental task. For the reference of interested readers, a history of camera models and correction techniques, largely driven by aerial photography for map-making can be found in (Clarke and Fryer, 1998). A survey for the prevailing calibration and orientation techniques can be found in (Gruen and Huang, 2001) and (Remondino and Fraser, 2006). In this section, a brief review of some recent research works is provided.

Over the past century, numerous methods ranging from rigorous laboratory-conducted calibration to image based auto-calibration have been applied for retrieving the intrinsic camera parameters, which may include focal length, principle point location, skew and lens distortions. In computer vision, the calibrating techniques are even more diverse. For example, a plane-based approach was proposed for calibrating a fish-eye lens camera (Li and Hartley, 2006). A multi-view geometry of 1D radial cameras is applied for the omnidirectional camera calibration in (Thirthala and Pollefeys, 2005). For general cameras, a specific rational function lens distortion model is developed by (Claus and Fitzgibbon, 2005). Among various sensor models, in this paper we mainly focus on consumer-grade digital cameras that have no extreme distortions.

The most commonly adopted methods for internal camera calibration include observing a planar pattern shown at a few different orientations (Zhang, 1999; Gurdjos and Sturm, 2003) or factorization of homography matrices (Ueshiba and Tomita, 2003). The calibration function can also be determined directly from views captured by the camera without any special knowledge of the scene, and one of the fastest algorithms is described in (Manning and Dyer, 2003). Another approach of image-based camera calibration that exploits the information inherent in vanishing lines has emerged and become popular in the recent years. Original approaches used three vanishing lines (Wang and Tsai, 1991), and then with the goal of making the approach more practical, the minimum requirement for this method was reduced to two vanishing lines (Grammatikopoulos et al., 2004).

External camera calibration refers to determining the position and orientation of an internally calibrated camera relative to the world reference frame. In (Triggs, 1999), the camera orientation is estimated using quasilinear methods. Apart from the traditional point-based approaches, some research efforts have been made to determine the camera orientation by matching linear features. In (Schenk, 2004) and (Karjalainen et al., 2006), the typical collinearity model is modified for expressing orientation and tie line parameters as a function of points measured on image lines.

In this paper we will introduce a 4-point correspondence procedure. It utilizes the property of scale ratio for two image points introduced in (Lai and Yilmaz, 2008) for the estimation of focal length. The camera focal length is then incorporated with the geometry formed by camera center, principle point, vanishing point and vanishing line to find the three rotation angles about the world reference frame. The proposed approach provides a fast and direct solution to the task of finding camera parameters and the results can also be used as an alternative initial approximation for other rigorous methods.

2. ESTIMATION OF FOCAL LENGTH

In this section, we exploit the single-view geometry to estimate the camera focal length via recovering the vanishing point of a reference direction and the vanishing line of the planes orthogonal to the direction. Note that, in this paper the principle point is assumed to lie at the image center and the distortion effects are assumed to be negligible.

2.1 Projective geometry

The projection of a point $\mathbf{X}$ = [X Y Z 1]^T in the object space to a point x = [x y 1]^T in the image space is expressed in terms of a direct linear mapping in the homogeneous coordinates as:

$$\lambda\mathbf{x} = \mathrm{P}\mathbf{X} \tag{1}$$

in which λ is the scale factor due to the projective equivalency $\lambda\mathbf{x} = \mathbf{x}$, and P is the camera projection matrix. In the case when the imaged points are lying on a plane in the object space, the projection matrix given in equation (1) reduces to plane projective transform. Without the loss of generality, if we choose the reference plane in the scene as the "ground plane" which means Z = 0 and let $\mathbf{p}_i$ denotes the i_{th} column vector of P, the linear mapping given in (1) reduces to the homography transform

$$s\mathbf{x'} = \mathrm{H}\mathbf{X'} = [\ \mathbf{p}_1\ \mathbf{p}_2\ \mathbf{p}_4\ ]\ [\ X\ Y\ 1\ ]^T \tag{2}$$

where $\mathbf{x'}$ is the image of point $\mathbf{X'}$ on the X-Y plane and H is the homography matrix. This formulation introduces another scale factor s, which is imposed to ensure that the last element of homogeneous coordinate vector is equal to 1.

The projection in (1) can also be represented using vanishing points. When projected to the image plane, a pair of parallel lines in the object space intersects at a single point called the vanishing point. By using the inherent property that column vector $\mathbf{p}_3$ of P corresponds to $\mathbf{v}_z$, the vanishing point of Z axis, the projection in (1) can be rearranged as:

$$\lambda\mathbf{x} = s\mathbf{x'} + Z\mathbf{v}_z \tag{3}$$

In equation (3), λ = s + Z due to the setting that the last element of any homogeneous coordinate vector is 1. If both $\mathbf{x'}$ and $\mathbf{x}$ can be identified on the image and Z is known for point $\mathbf{X}$, s is trivial to compute.

2.2 Estimating focal length from scale ratio

A vanishing line is the image of an ideal line lying at infinity, and it can be determined intuitively from two vanishing points. In (Lai and Yilmaz, 2009) the vanishing line of X-Y plane is estimated from the property of scale ratio (Lai and Yilmaz, 2008):

Scale ratio of two image points: *When projecting two points $\mathbf{X}_1$, $\mathbf{X}_2$ lying on a plane π in the object space onto the corresponding points $\mathbf{x}_1$, $\mathbf{x}_2$ in the image space using homography, the ratio of the introduced scale factors s_1 and s_2 is the inverse proportion of the distances from the image points to the vanishing line of plane π.*

Let D($\mathbf{l}_v$, $\mathbf{x}_i$) denote the distance from image point $\mathbf{x}_i$ to the vanishing line $\mathbf{l}_v$, the property can be expressed as

$$\frac{s_1}{s_2} = \frac{D(\boldsymbol{l}_v, \boldsymbol{x}_2)}{D(\boldsymbol{l}_v, \boldsymbol{x}_1)} \tag{4}$$

Estimating focal length, f, from the scale ratio and vanishing line has been introduced in (Lai and Yilmaz, 2009). In this paper, we propose an alternative derivation without explicitly solving for the vanishing line. By assuming that the principal point coincides with the image center, the image points can be centered and calibration matrix simplified to K = diag(f, f, 1). Let ω = diag(1/f^2, 1/f^2, 1) be the image of the absolute conic. Due to the orthogonality in the object space, the vanishing point $\mathbf{v}_z$ and the vanishing line $\mathbf{l}_v$ are related by

$$\mathbf{l}_v = \omega\ \mathbf{v}_z \tag{5}$$

More detail about this relationship can be found in (Hartley and Zisserman, 2004). Let $\mathbf{v_z} = [v_1\ v_2\ 1]^T$, $\mathbf{x_1} = [x_1\ y_1\ 1]^T$ and $\mathbf{x_2} = [x_2\ y_2\ 1]^T$. By replacing ω with the values of f and using equation (4), we obtain directly the equation:

$$f^2 = (s_2 (v_1x_2 + v_2y_2) - s_1 (v_1x_1 + v_2y_1)) / (s_1 - s_2) \qquad (6)$$

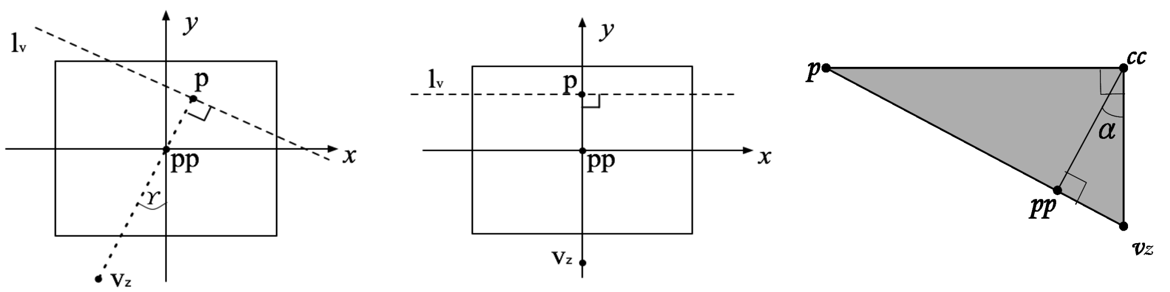
Hence, if two points on a plane are identified on the image and their scale ratio and the vanishing point of the normal to the plane are computed, camera focal length can be obtained with only four points measurements. We have to emphasize here that the focal length computed from (6) is an approximation and not a rigorous calibration result. Also, the scale ratio is independent of the actual length of the measured linear feature in the object space, only the relative length or height is required. Instead of six-point correspondences that other approaches require to recover the projection matrix, the proposed approach utilizes information inherent in these four points for retrieving camera rotations relative to a plane, as will be elaborated in the next section.

3. CAMERA ROTATION RELATIVE TO A PLANE

A variety of approaches have focused on the recovery of camera rotation about a world coordinate frame. Instead of providing robust camera pose estimation, in this section we demonstrate a technique for intuitively and flexibly aligning an image plane relative to a plane π in object space. Assume focal length f, vanishing line $\mathbf{l_v}$ of a plane π and vanishing point $\mathbf{v_z}$ of the plane normal are obtained using the approach described in the previous section. The image x and y axes are assigned as shown in Figure 1(a) and z coincides with the principle axis. Let **pp** be the principle point and α, β, γ denote rotation angles about the camera x, y, and z axes, respectively. First the rotation is performed about z by

$$\gamma = \arctan\left(\frac{v_2}{v_1}\right) \ , \qquad (7)$$

after which $\mathbf{v_z}$ lies at y axis and $\mathbf{l_v}$ becomes parallel to the x axis (Figure 1(b)).



(a) (b) (c)

Figure 1. (a) Geometry between $\mathbf{l_v}$, $\mathbf{v_z}$ and **pp** in the original image. (b) Geometry after first rotation. (c) Geometry between $\mathbf{l_v}$, $\mathbf{v_z}$, **pp** and camera center **cc**.

The next rotation depends on whether the image plane is to be aligned as parallel to the plane π or to the plane normal. According to the geometry shown in Figure 1(c), in the first case the image is rotated about x by α' = - α where

$$\alpha = \arctan\left(\frac{D(v_z, pp)}{f}\right) \qquad (8)$$

and $\mathbf{v_z}$ is moved to the location of **pp** after rotation. If the rotation angle is α' = π/2 – α instead, $\mathbf{l_v}$ lies at x axis and $\mathbf{v_z}$ is cast to infinity as expected for the second case.

Following a similar rotation procedure, the image axis can be aligned to any line **l** parallel to π in object space using the rotation about y axis. Let **l** intersect $\mathbf{l_v}$ at $\mathbf{v_x}$ in the original image. After the rotations by γ and α' = π/2 – α, $\mathbf{v_x}$ is transformed to its new location

$$v_x' = K R_x R_z K^{-1} v_x \qquad (9)$$

where the rotation matrices are

$$R_z = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha' & -\sin\alpha' \\ 0 & \sin\alpha' & \cos\alpha' \end{bmatrix}. \quad (10)$$

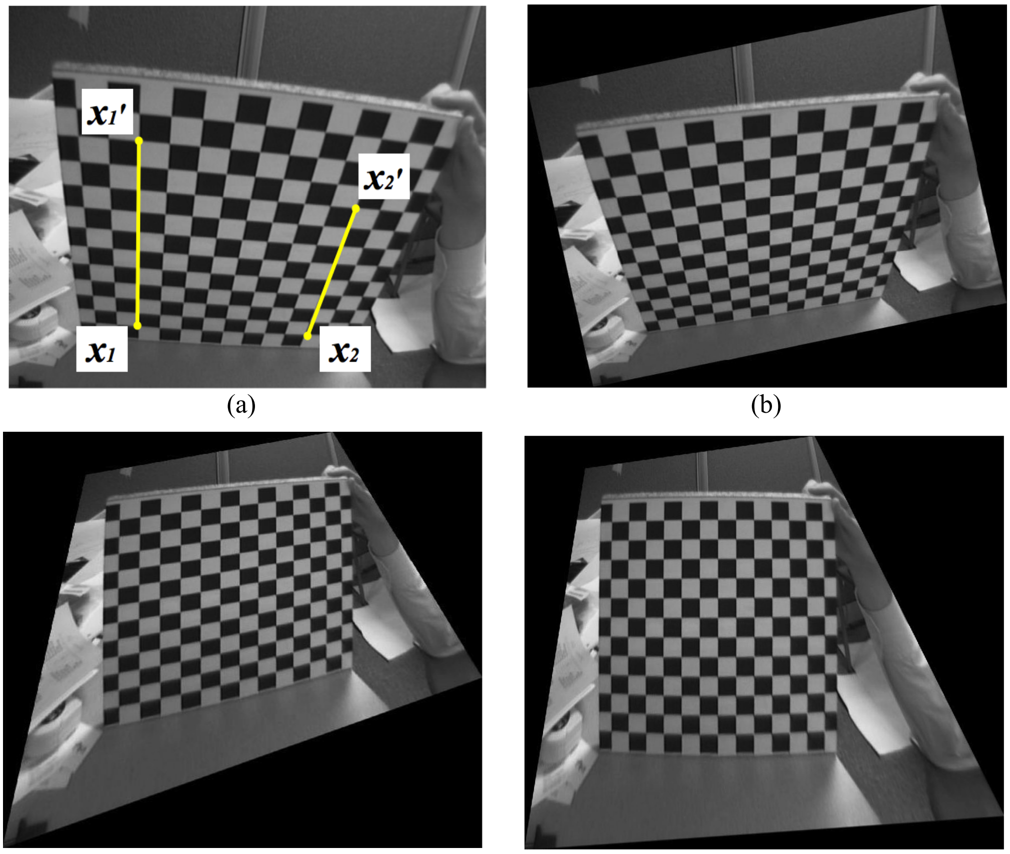
Again, when the image plane is rotated about the y axis by

$$\beta = \arctan\left(\frac{D(v_x', pp)}{f}\right), \text{ and } R_y = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}, \quad (11)$$

vx' is moved to infinity and image plane is now parallel to **l**. This process enables the estimation of angles between specific planes or linear features and can be extended into other applications such as image rectification, which is demonstrated in the next section.

4. EXPERIMENTS

The first experiment used an image of the calibration pattern and its calibrated focal length provided in (Bouguet, 2010) to demonstrate the 4-point correspondence procedure. In the original image (Figure 2(a)), assume two points $\mathbf{x}_1$ and $\mathbf{x}_2$ correspond to the points lying on a line parallel to the X axis in the object space. The point pairs ($\mathbf{x}_1$, $\mathbf{x}_1'$) and ($\mathbf{x}_2$, $\mathbf{x}_2'$) provide the vanishing point $\mathbf{v}_z$. In addition, the heights (or Z values) for $\mathbf{x}_1'$ and $\mathbf{x}_2'$ are set as 100 and 80 respectively. The focal length f = 657.7 is computed using the steps described in Section 2, and this value is very similar to the rigorously calibrated focal length value f = 657.5 .

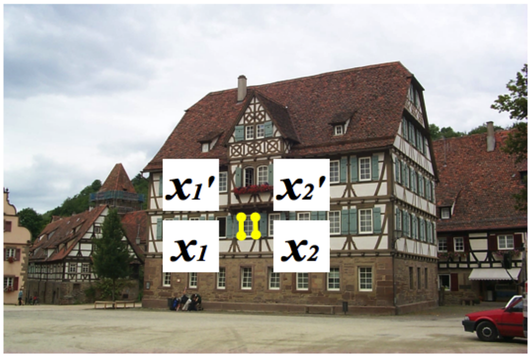


(a) (b) (c) (d)

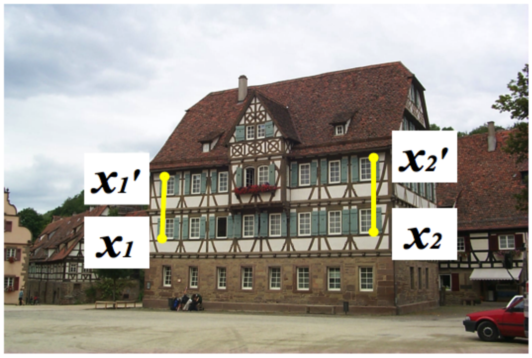
Figure 2. Experiment using calibration pattern: (a) Original image and the four points used for computing scale ratio. (b) Transformed image after rotating γ about the z axis. (c) Transformed image after rotating π/2 – α about the x axis. (d) Rectified image after rotating β about the y axis.

Although in this example, image rectification of the X-Z plane can be achieved directly by transforming original image I as $I' = K R_y R_x R_z K^{-1} I$, the image after each rotation is included in Figure 2 to illustrate each step in the process. It is clearly observed that the grid pattern is recovered as squares in the last image.

Another experiment on the image provided in (ISPRS, 2008) helps reveal the validity of applying the proposed approach on real world scenes. As shown in Figure 3(a), one of the window panes near image center is selected and measured for computation. The computed focal length f = 1954.2 is significantly greater than the calibrated value f = 1736.7. When another set of points with a larger span is selected (Figure 3(b)), the computed focal length f = 1615.7 is much closer to the calibrated value. This is because the focal length estimation is sensitive to error in the calculated vanishing point location. If the vanishing point is obtained from points with more concentrated spatial distribution (shorter distance between points), its location is more affected by the measurement error, hence the estimated focal length is less reliable. In addition, the approximation is obtained from only four points and distortions are not modeled. However, this approach can still provide a fast way to generate an initial guess of parameters that can be utilized in other robust calibration approaches.



(a)



(b)

(c)

(d)

(e)

Figure 3. Experiment on a building image: (a) Only one window is used for computing vanishing point and scale ratio. (b) Four points with larger spans are selected. (c) Transformed image after rotating γ about the z axis. (d) Transformed image after rotating π/2 – α about the x axis. (e) Rectified image after rotating β about the y axis.

5. CONCLUSION

We have presented a new approach for the estimation of camera parameters by exploiting the characteristics of parallelism and orthogonality in projective geometry. The proposed approach provides flexible and fast estimations for camera focal length and rotations from a single vanishing point and scale ratio, and is useful especially in the cases where it is hard or impossible to perform rigorous calibration due to limited information about object space.

REFERENCES

Bouguet, J. Y., 2010. Camera Calibration Toolbox for Matlab, http://www.vision.caltech.edu/bouguetj/calib_doc/index.html.

Clarke, T. A. and Fryer, J. G., 1998. The development of camera calibration methods and models. The Photogrammetric Record, 16(91), pp. 51-66.

Claus, D. and Fitzgibbon, A. W., 2005. A rational function lens distortion model for general cameras. In: Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition, Vol.1, pp. 213-219.

Grammatikopoulos, L., Karras, G. and Petsa, E., 2004. Camera calibration combining images with two vanishing points. International Archives of Photogrammetry, Remote Sensing and Spatial information Sciences, 35(5), pp. 99-104.

Gruen, A. and Huang, T. S., 2001. Calibration and Orientation of Cameras in Computer Vision. Springer.

Gurdjos, P. and Sturm, P., 2003. Methods and geometry for plane-based self-calibration. In: IEEE Computer Society Conference on Computer Vision and Pattern Recognition, Vol.1, pp. 491-496.

Hartley, R. and Zisserman, A., 2004. Multiple View Geometry in computer Vision, 2nd edition. Cambridge University Press.

ISPRS, 2008. ISPRS Data sets: Old Building, http://www.isprs.org/data/oldbuilding/default.aspx.

Karjalainen, M., Hyyppa, J. and Kuittinen, R., 2006. Determination of exterior orientation using linear features from vector maps. The Photogrammetric Record, 21(116), pp. 329-341.

Lai, P. and Yilmaz, A., 2008. Efficient object shape recovery via slicing planes. In: Proceedings of IEEE Conference on Computer Vision and Pattern Recognition.

Lai, P. and Yilmaz, A., 2009. A new approach for vanishing line estimation. In: Proceedings of American Society for Photogrammetry and Remote Sensing Annual Conference.

Li, H. and Hartley, R., 2006. Plane-based calibration and auto-calibration of a fish-eye camera. In: Proceedings of 7th Asian Conference on Computer Vision, pp. 21-30.

Manning, R. A. and Dyer, C. R., 2003. Research on self calibration without minimization, Technical Report No. 1490, Department of Computer Sciences, University of Wisconsin.

Remondino, F. and Fraser, C., 2006. Digital camera calibration methods: considerations and comparisons. International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences, XXXVI(5), pp. 266-272.

Schenk, T., 2004. From point-based to feature-based aerial triangulation. ISPRS Journal of Photogrammetry and Remote Sensing, 58(5-6), pp. 315-329.

Thirthala, S. and Pollefeys, M., 2005. Multi-view geometry of 1d radial cameras and its application to omnidirectional camera calibration. In: Proceedings of the 10th IEEE International Conference on Computer Vision, pp. 1539-1546.

Triggs, B., 1999. Camera pose and calibration from 4 or 5 known 3d points. In: Proceedings of 7th International Conference on Computer Vision, pp. 278-284.

Ueshiba, T. and Tomita, F., 2003. Plane-based calibration algorithm for multi-camera systems via factorization of homography matrices. In: Proceedings of 9th IEEE International Conference on Computer Vision, Vol.2, pp. 966-973.

Wang, L. and Tsai, W., 1991. Camera calibration by vanishing lines for 3-d computer vision. IEEE Transactions on Pattern Analysis and Machine Intelligence, 13(4), pp. 370-376.

Zhang, Z., 1999. Flexible camera calibration by viewing a plane from unknown orientations. In: Proceedings of the 7th IEEE International Conference on Computer Vision, Vol.1, pp. 666-673.