Using Iterative Weighting to Fit the Circular Parameters of Vertical Cylinder in Point Clouds

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ABSTRACT: Most spatial data collection and extraction have transited from field work to office work by surveying and mapping technical innovation. More and more studies focus on the 3D object modeling based on automatic and semi-automatic modeling algorithms. Especially, the point clouds with highly density and precision collected from LIDAR (light detection and Ranging) are beneficial for 3D modeling by different algorithms. Therefore, the objective of this study is to develop the fitting algorithms to determine the circular parameters of the vertical cylinders of streetlamps or traffic poles in road environment from the point clouds. Unfortunately, two reasons make the fitting more difficult due to the real scanning situations. The first one is incomplete scanning because of the scanning position. Therefore, the point clouds cannot cover all the surface of the vertical cylinders. The second one is that not all the point clouds are from surface of the vertical cylinders, part of them are reflected from other attached objects, such as advertising flags or traffic sign devices. To cope with the second problem, least square with iterative weighting is used in this study to overcome the difficulties that we just mentioned about. Finally, the fitting results will be discussed for 3D modeling of the vertical cylinders in the future study.

1. INTRODUCTION

Using the laser scanning system makes point clouds collection of field works accomplished in short time with high efficiency. The point cloud data have two characters: high-precision and high-density. More and more studies, such as traffic environment modeling, 3D city modeling and pipeline allocation modeling in the factory, focus on the automatic 3D modeling algorithms. Poles, like the part of traffic lights and signs, are the main structure in traffic environment and most of them are constructed by cylinders. Major studies used to fitting the parameters of cylinders, i.e. radius and center point, by Least Squares Fitting. Nevertheless, there are some attachments to cylinders, e.g. flags or traffic devices. Additionally, the collected points just cover part of cylindrical surface. These two situations could cause the difficulty to fitting the parameters of cylinders. This study use iterative weighting of Least Square Fitting to cope with these problems. In the iterative weighting process of Least Squares Fitting, the weights of the observations would change depending on weight functions which are concerned with the residuals of the observations. By the proposed method, the wrong observations will be removed and the parameters will be determined correctly. Section 2 will describe the proposed methodology and the test results will be presented and discussed in section 3. Finally, we will give our conclusions and suggestions for future study.

2. METHODOLOGY

Traditionally, all the observations are regarded as same weight when using the least squares adjustment. Once the mistakes exist in observations, the impact on the adjustment results is directly proportional to the amount of errors in observations. Therefore, least square method only applies to observations with random errors, and it does not apply to the observations with gross errors (Klein and Foerstner, 1984). To fit the circular parameters of vertical cylinders, the wrong points in the point clouds should be removed. This study proposed the iterative weighting of Least Squares Fitting to lower the weight of the wrong observations. Every observation is given different weight according to its residual in the preivious least squares fitting. To lower the weight of the observation with mistake, this study chose the iterative weighting approach. The general idea of iterative weighting is to choose a right weight function and to make the weights of wrong observations approach to zero when iterating fitting converges. Therefore, the wrong observations would not participate in the adjustment (Kubik et al., 1987). Then we can acquire the correct circular parameters of vertical cylinders. Figures 2.1 and 2.2 illustrate the iterative weighting fitting from the results of the straight line fitting by using equal weight fitting and iterative weighting fitting respectively.

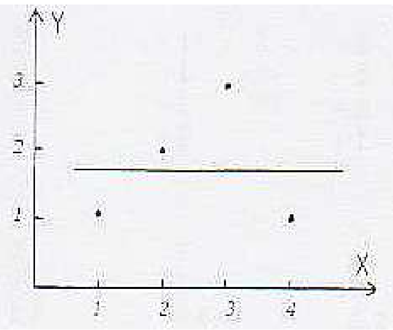

Figure2.1 equal weight fitting of the straight line

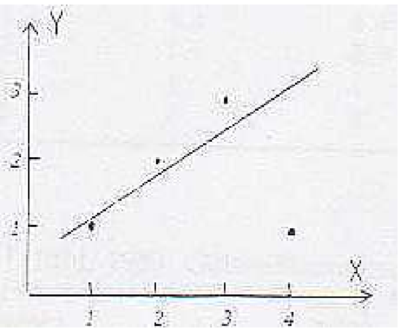

Figure2.2 iterative weighting fitting of the straight line

Figures source：(Kubik et al.,1987)

How to determine the appropriate weight function is extremely important work. According to Kubik and Merchant (1988), the weight function determined by Danish method is outstanding for efficiency of convergence and good results for robust estimation. Therefore, this study will use Danish method for iterative weighting to fit the circular parameters of the vertical cylinders. The weight function will decrease the weights of the wrong observations when the residuals of the observations increase based on the formula (1). After the iterative convergence, because the weights of the wrong observations would be extremely small, the wrong observations would not affect the result of adjustment.

$$p^{(v+1)} = p^{(v)} f(v)$$

$$f(v) = \begin{cases} 1, & \frac{|v|\sqrt{P_0}}{cm_0} < c \\ \exp\left(\frac{-|v|\sqrt{P_0}}{cm_0}\right), & \frac{|v|\sqrt{P_0}}{cm_0} \geq c \end{cases} \tag{1}$$

$p^{(v)}$: weight function $f(v)$: residual function v: residual of observation c: constant=3
P_0： coefficient of weight m_0: standard deviation of observation

When the point clouds are used to fit the circular parameters of the vertical cylinders, the observations are the X and Y coordinates. The formula of circle is nonlinear, see formula (2). Therefore ,formula (2) should be processed by Taylor series linearization, see formula (3). After linearization, the formula (3) can be expressed by matrix method, like formula (4).

$$(X_i - X_0)^2 + (Y_i - Y_0)^2 - r^2 = 0 \tag{2}$$

Xi, Yi: point X and Y coordinates X_0 ,Y_0: circle center r:circule radius

$$2[(X_i - X_0)V_{Xi} - (X_i - X_0)dX_0 + (Y_i - Y_0)V_{Yi} - (Y_i - Y_0)dY_0 - rdr] = F_0 \tag{3}$$

dX_0 , dY_0: increment of the circular center dr: radius increment F_0: function model of observation
Xi, Yi: point X and Y coordinates V_{Xi} , V_{Yi}: residuals of point X and Y coordinates

$$AV + B\square + f = 0 \tag{4}$$

A: coefficient matrix of function which is linearization V: residual vector of observations
B: coefficient matrix of the estimated parameters Δ: vector of parameter increment
f: difference between f(Xn, Yn, Zn) and f(Xn+1,Yn+1,Zn+1)

According to least squares principle complying with formula (4), formula (5) will be derived, where $N= B^T(AW^{-1}A^T)^{-1}B$ and $T = -B^T(AWA^T)^{-1}f$. Then, $\Delta=N^{-1}T$ can be applied to get dX_0, dY_0 and dr when iterative fitting converges.

$$B^T\left(AW^{-1}A^T\right)^{-1}B\square = -B^T\left(AWA^T\right)^{-1}f \tag{5}$$

As for the initial values of the fitting, they were acquired by manual method in the Phidias software. In the first iteration, the weights of observations depend on the radius differences. The radius difference means the difference between distance from point to the initial center of the circle and initial radius. Using negative absolute of the radius difference as the coefficient of power of exponent, the radius difference which is larger than a certain critical value makes the weight be close to zero. The weight based on the radius difference is to make sure the fitting circular parameters in the first iteration still correct for the follow-up iterative fitting. The certain critical value is 2cm in this study.

In the follow-up iteration, the weights of observations are according to two criteria. One is the radius difference, the other one is the observation residual. The radius difference, the same as the first iteration, means the difference between the distance from point to the fitting center of the circle and the fitting radius by iterative weighting fitting. The adopted weight function is the same as the first iteration. The second kind of weigh function is based on the calculation of the each observation residual. From formula (1), the weight of observations can be calculated, where m0 is 5 mm in this study. One point contains X and Y coordinates and every X and Y coordinates has its own residual. This situation would make the weights of X and Y different. In common sense, if a point is a wrong observation, not only X but also Y coordinate is wrong. Therefore, we should make the weights of X and Y coordinates equal.

3. TESTS AND RESULTS

3.1 Instrument Description

This study uses the Leica HDS 3000, the instrument specifications is shown as Table 3.1. The scan mode is high space resolution (mm) when the point cloud is acquired. Two test datasets are: (1). a streetlamp near the school gate of Chengchi University; and (2). a billboard in Chengchi University. The photos of these two test datasets are shown in Figures 3.1 and 3.2, respectively.

Table 3.1 Instrument Specifications

brand model	Leica HDS 3000	horizontal field-of-view	360°
scan mode	Push broom	vertical field-of-view	270°
wavelength	532nm	distance accuracy	4mm@50m
angle accuracy	0.0034 radian	positional accuracy	6mm@50m


Figure3.1 photo of streetlamp near school gate


Figure3.2 photo of billboard in school

These two datasets contain cylindrical point cloud and non-cylindrical point cloud. In this study, the part of these datasets is selected for two study objects. Study design is towards two directions. The first one focuses on the fitting results from different weight function. The second one will focus on the affection of fitting results from different ratios of non-cylinderical points, which is corresponding to the blunder ration. The blunder ratios are 95%, 90%, 50%, 40%, 30%, 20%, and 0%, respectively.

3.2 Test Results

3.3.1 The fitting results from different weight function

As mentioned in Section 2, the weights of observations depend on the radius differences in the first iteration to exclude the wrong points from the fitting adjustment as possible as it can be. After that, the follow-up iterations, the weights of observations are determined by (1) the radius differences；(2) the observations(Obs.) residuals. The fitting results from Test dataset 1 are shown as Table 3.2, where blunder ratio means the ratio of non-cylindrical points in the whole dataset.

Table3.2 Fitting results of different weight function from Test dataset 1

Blunder ratio	Weight function based on	Iteration numbers	Fitting result(unit: meter)			Difference(unit: meter)		
			X	Y	r	△X	△Y	△r
95%	Radius differences	5	52.916	-28.702	-0.286	0.336	0.030	0.353
	Obs. residuals	4	53.239	-28.666	0.060	0.013	-0.006	0.007
90%	Radius differences	5	52.916	-28.702	-0.287	0.336	0.030	0.354
	Obs. residuals	4	53.240	-28.666	0.061	0.012	-0.006	0.006
50%	Radius differences	22	52.939	-28.750	-0.288	0.313	0.078	0.355
	Obs. residuals	4	53.25	-28.672	0.066	0.002	0.000	0.001
40%	Radius differences	15	52.942	-28.730	0.278	0.310	0.058	-0.211
	Obs. residuals	4	53.25	-28.672	0.066	0.002	0.000	0.001
30%	Radius differences	8	53.208	-28.687	0.044	0.044	0.015	0.023
	Obs. residuals	4	53.250	-28.672	0.066	0.002	0.000	0.001
20%	Radius differences	5	53.219	-28.683	0.051	0.033	0.011	0.016
	Obs. residuals	4	53.250	-28.672	0.066	0.002	0.000	0.001
0%	Radius differences	4	53.252	-28.672	0.067			
	Obs. residuals	4	53.252	-28.672	0.067			

From Table 3.2, there is no difference between fitting results from different weight function when the blunder ratio is 0%. The differences between fitting results and the manual measurement from the Test dataset 1 with blunder ratio 0% are less than 1 cm. Therefore, these fitting results are regards as reference value for comparison. The fitting results of iterative weighting based on observation residuals are getting better and better than those fitting results of iterative weighting based on radius difference when the blunder ratio is increasing. Figures 3.3 and 3.4 shown the fitting results from different weight function where the blunder ratio is 40%. From Figure 3.4, because part of non-cylindrical point observations in the iterative weighting fitting based on radius difference participate in the fitting adjustment, the fitting results don't fit to cylinder in reality.

Figure 3.5 illustrates the 3D modeling of fitting results from different weight function by Phidias software, where the blunder ratio is 40%. The white cylinder wireframe is modeled by manual operation from point cloud directly. The red cylinder wireframe is modeled from fitting result whose weight depends on radius difference. The green cylinder wireframe is modeled from fitting result whose weight depends on observations residuals. The difference of X-Y plane between white wireframe and green wireframe is 1.5 cm. The difference of X-Y plane between white wireframe and red wireframe is 52.7 cm. The fitting result whose weight depends on observation residual is more suitable for cylinder modeling in reality. Therefore, the weight function based on observations residuals is used for the following test.

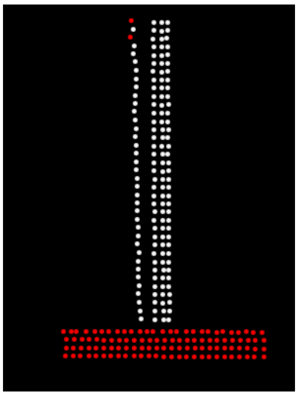

Figure3.3 Fitting result of weight from obs. residuals

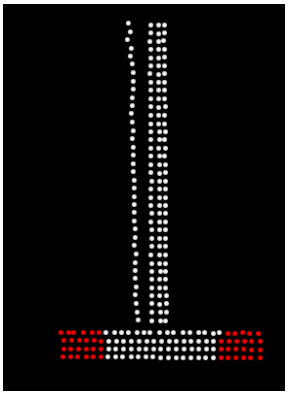

Figure3.4Fitting result of weight from radius difference

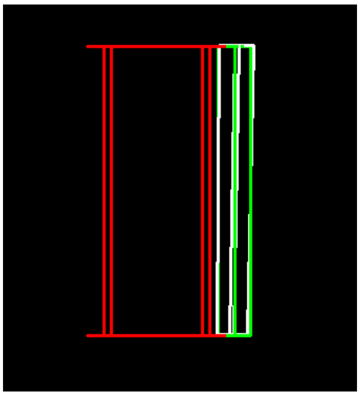

Figure3.5 3D modeling of different fitting results, where blunder ratio is 40%

(white point: cylindrical points Red point: non-cylindrical points)

3.3.2 Bundle ratio

The following test will use the weight function based on observation residuals after the second iterative weighting fitting.

(1) Test dataset 1: Point cloud of Streetlamp near school gate

As mentioned in Section 3.1, the study target for test dataset 1 is the streetlamp near school gate of Chengchi University. As shown in Table 3.2, the fitting result with blunder ration 0% is circular center (53.252m,-28.672m) and the radius 0.067m.The differences between fitting results and the manual measurement from dataset 1 with blunder ratio 0% is less than 1 cm. Therefore, this fitting result is regards as reference value for comparison. The non-cylindrical points are not directly adhered to the cylinder in Test dataset 1. The proposed fitting method would remove these points according to the weighting fitting based on the radius difference in the first iteration. Figure 3.6 shows the fitting result with blunder ration 40 %. Even the blunder ratio is 95%, the proposed fitting method can recognize the cylindrical points and non-cylindrical points correctly, as shown in Figure 3.7. In Figures 3.6 and 3.7, the red points mean non-cylindrical points. The white points are cylindrical points. The 3D modeling results of fitting parameters are used to verify the results by visual approach, see Figure3.9. The white wireframe is modeling by manual measurement from point cloud directly. The green wireframe is modeling based on the fitting result with blunder ration 40%. The difference between white wireframe and green wireframe is less than 1.5 cm.

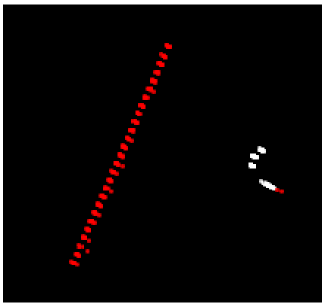

Figure3.6 Fitting result with blunder ratio: 40%

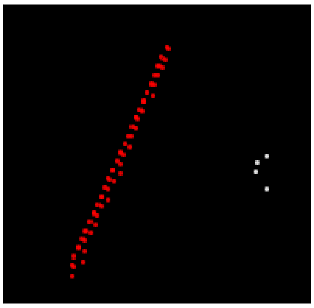

Figure3.7 Fitting result with blunder ratio: 95%

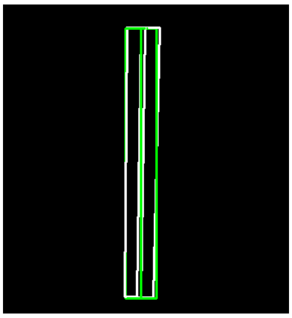

Figure3.8 3D modeling of different fitting results with blunder ratio is 40%

(2) Test dataset 2: Billboard in in Chengchi University

Table 3.3 Fitting result with different blunder ratio from test dataset 2

Blunder ratio	Iteration numbers	Fitting result(unit: meter)			Difference(unit: meter)		
		X	Y	r	△X	△Y	△r
95%	6	-7.050	-1.129	0.078	-0.084	0.014	-0.052
90%	6	-7.050	-1.133	0.074	-0.084	0.018	-0.048
50%	8	-7.056	-1.144	0.063	-0.078	0.029	-0.037
40%	2	-7.129	-1.122	0.024	-0.005	0.007	0.002
30%	2	-7.134	-1.118	0.026	0.000	0.003	0.000
20%	2	-7.133	-1.116	0.025	-0.001	0.001	0.001
0%	2	-7.134	-1.115	0.026			

As shown in Table 3.3, the fitting result with blunder ratio 0% is circular center (-7.134m,-1.115m) and the radius 0.026 m. The differences between fitting results and the manual measurement from dataset 2 with blunder ratio 0% is less than 1 cm. Therefore, this fitting result is also regards as reference value for comparison.

In this test dataset, the non-cylindrical points of billboard are adhered to the cylinder. We can’t use the first iterative weighting based on the radius difference to remove the wrong points. The fitting result will be not better even though the threshold of the radius difference is reduced. From Table 3.3, the fitting results with blunder beyond 40% are not acceptable. The fitting results with blunder ratio 40% are shown in Figures 3.9 and 3.10. The red points mean non-cylindrical points. The white points are cylindrical points. The 3D modeling approach of fitting results is used to verify the results by visual approach, see Figure3.11. The white wireframe is modeling by manual measurement from point cloud directly. The green wireframe is modeling based on the fitting result with blunder ratio 40%. The difference in X-Y plane between white wireframe and green wireframe is less than 1.3 cm.

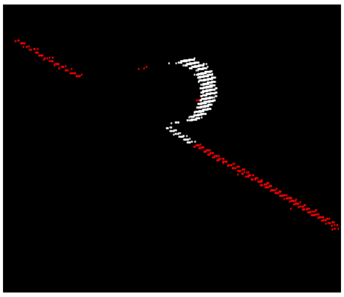

Figure3.9 Fitting result which blunder ratio is 40% in top view

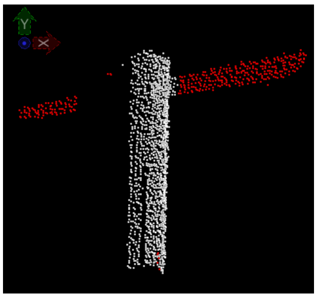

Figure3.10 Fitting result which blunder ratio is 40% in isometric view

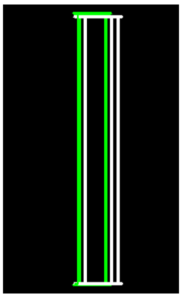

Figure3.11 3D modeling with blunder ratio is 40%

4. CONCLUSIONS AND SUGGESTION

According to the test results, our proposed fitting algorithm can remove the wrong points from fitting when the non-cylindrical points are not close to the cylinder. The wrong points, blunder ratio beyond 40%, cannot be removed totally when the non-cylindrical points are adhered to the cylinder. Additionally, the following conclusions and suggestions will be drawn.

(1) Circular parameters for non-vertical cylinder

In reality, most cylinders are not vertical. The circular parameters of the non-vertical cylinders should be ellipse parameters for more correct fitting results.

(2) Size of the cylinder

Most of the cylinders have smaller radius in the top and larger radius in the bottom, such as streetlamp in traffic environment. If much more detailed 3D modeling is requested, the size of the cylinders should be considered.

(3) Initial values

Because the circular equation is non-linear, the initial values of circular function are necessary to linearize the circular equation for correct iterative fitting. In this study, the initial values are acquired by manual measurement. How to get a good initial value automatically is most concerned in future study.

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