COMPARISON OF RESIDUAL GEOID HEIGHT MODELING USING LSC AND FFT METHODS IN KOREA

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ABSTRACT: In this study, we performed the residual geoid modeling using the FFT and LSC methods in context of application of R-R (Remove and Restore) technique as a general technique for gravimetric geoid model in order to propose the effective way of geoid determination in Korea. For this, a number of data compiled for residual geoid modeling by the multi-band spherical FFT method with Stoke's formula and LSC method as known as statistical method. The geometric geoidal heights obtained from 503 GPS/Levelling data were used for inducing the various elements and proper computation process which should be considered for improving the accuracy of residual geoid modeling. Finally, we statistically compared the results of residual geoid heights between FFT and LSC methods and reviewed then the proper way of residual geoid modeling to the region of Korea. As the results of comparison, LSC method is not suitable for residual geoid modeling in Korea due to the noise and lack of gravity observations and the effects of local characteristics.

1. INTRODUCTION

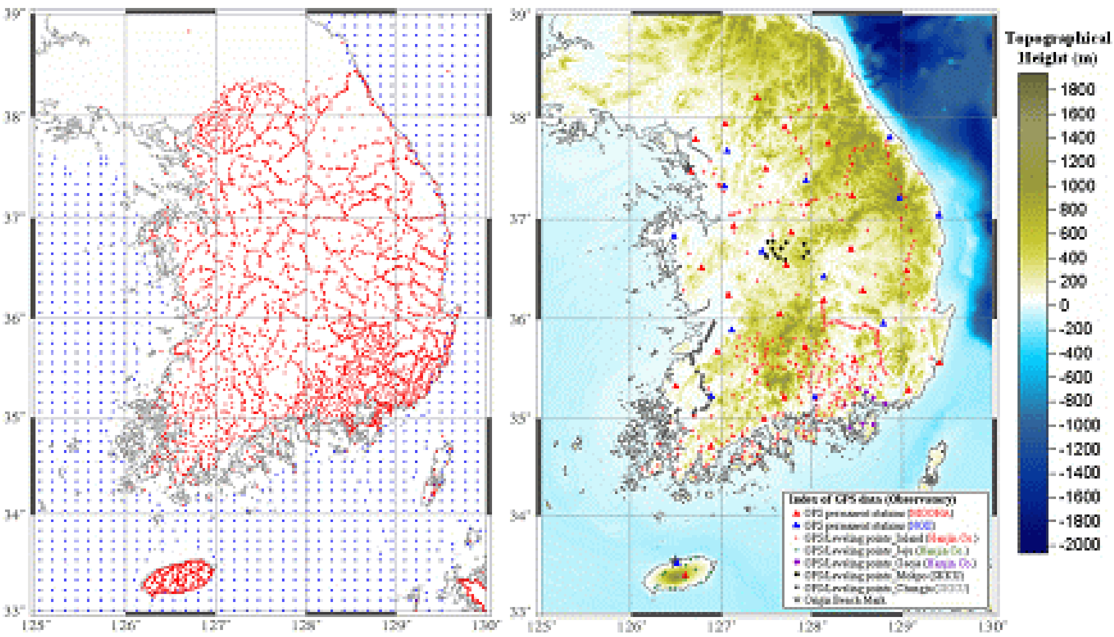
In general, a precise gravimetric geoid model is determined by accurately determining the long wavelength effect of geoid which is obtained through analysis of the earth's gravity field model, medium wavelength effect obtained through analysis of gravity anomaly between the earth's gravity field model and the actual gravity measurement, and the short wavelength effect obtained through terrain, and to synthesizing those effects (Yoon Dong Sik and Lee Dong Ha, 2005). Such a process of determining gravimetric geoid model is generally represented by Remove-Restore (R-R) technique, which is a method to calculate geoid dividing the effect into long wavelength, medium wavelength and short wavelength effects mentioned above, and to calculate precise geoid height (N) of a specific region by properly synthesizing the effect of each wavelength (Omang and Forsberg, 2000).

Efficient application of R-R technique for geoid modeling depends on use of more accurate earth's gravity field model and terrain data (DEM), selection of a gravimetric conversion method which is adequate for the region, and application of an effective modeling method for computation of the residual geoid height, and, at this time, modeling of residual geoid height which is medium wavelength effect of geoid come to have a big influence on the accuracy of gravimetric geoid (Lee Dong Ha, 2008).

In general, the modeling process of residual geoid height is comprised of a gridding process through Strokes integration, Vening-Meinesz integration, FFT (Fast Fourier Transformation) method, LSC (Least Square Collocation) method, point-mass fitting and expansion of spherical or elliptic harmonics (Forsberg and Tscherning, 1997), and, in this study, we carried out the analysis by applying multi-band spherical FFT method (Forsberg and Sideris, 1993; Yoon Hong Sik and Lee Dong Ha, 2005) with Strokes formula and LSC method respectively to 6,296 gravity measurement data (refer to Figure 1) obtained on land and at sea in Korea, for more precise computation of residual geoid height.

The gravity measurement data used for the study is comprised of total 4,298 land gravity measurement data made by National Geographic Information Institute, French BGI (Bureau Gravimetrique International) and Sungkyunkwan University and total 1,998 ocean gravity measurement data which DNSC (Danish National

Space Center) calculated by processing Topex/Poseidon, ERS and Geosat data. In order to obtain the residual gravity anomaly by removing the effects of the earth's gravity field model and of diverse terrains from the gravity measurement data, we applied EGM 2008 model (Hwang Hak et al., 2009) which is the earth's gravity field model with ultra-high degree developed most recently and RTM (Residual Terrain Model) method (Forsberg, 1984; Lee Dong Ha et al., 2008) which is known as the most adequate method for the domestic terrain.



(a) Gravity measurement data (b) GPS/Levelling data

Figure 1. Distribution of data used for this study

Finally, in this study, the diverse computation factors and proper computation process which should be decided in order to improve accuracy of computation result when modeling the residual geoid height applying FFT method and LSC method to gravity data mentioned above were determined using geometric geoid height obtained from total 503 GPS/Levelling data, and then the residual geoid modeling method most suitable for determination of Korean geoid was established by statistically analyzing the residual geoid height calculated by each method.

2. RESIDUAL GEOID MODELING USING FFT METHOD

Multi-band spherical FFT is the latest method designed in order to resolve the problem in calculation of Stokes integration (Stokes, 1849) which is a traditional geoid modeling method. By applying FFT to the object region being divided into latitude of an equal interval, instead of applying it to the whole object region like in the case of spherical FFT method, a calculation result of compositive residual geoid height with practically no error can be obtained through linear interpolation (Forsberg and Sideris, 1993). However, as an adequate integration cap size or modification of Stokes kernel is required for performance of accurate multi-band spherical FFT, accurate determinations of such computation factors should be preceded (Forsberg and Featherstone, 1998).
In this study, EGM2008 model and RTM conversion method were selected and applied respectively as the optimum earth's gravity field model and gravimetric conversion method in order to obtain residual gravity anomaly to which multi-band spherical FFT method was applied. However, as elevation anomaly and calculated residual gravity anomaly are based on the surface of the earth when RTM conversion method is applied, Molodensky integration which calculates residual quasi geoid was applied instead of Stokes formula. Also, in order to determine the number of multi-bands, 4-band spherical FFT which provides optimum result for the whole area of Korea was applied based on Yun's (1999) study, and 100% zero padding was applied in order to avoid influence of periodic convolution when calculating FFT (Yoon Hong Sik and Lee Dong Ha, 2005).
In case local residual geoid is modeled applying Stokes formula, the integration cap size of Stokes formula or the degree of Stokes kernel should be appropriately considered without fail (Forsberg and Featherstone, 1998),

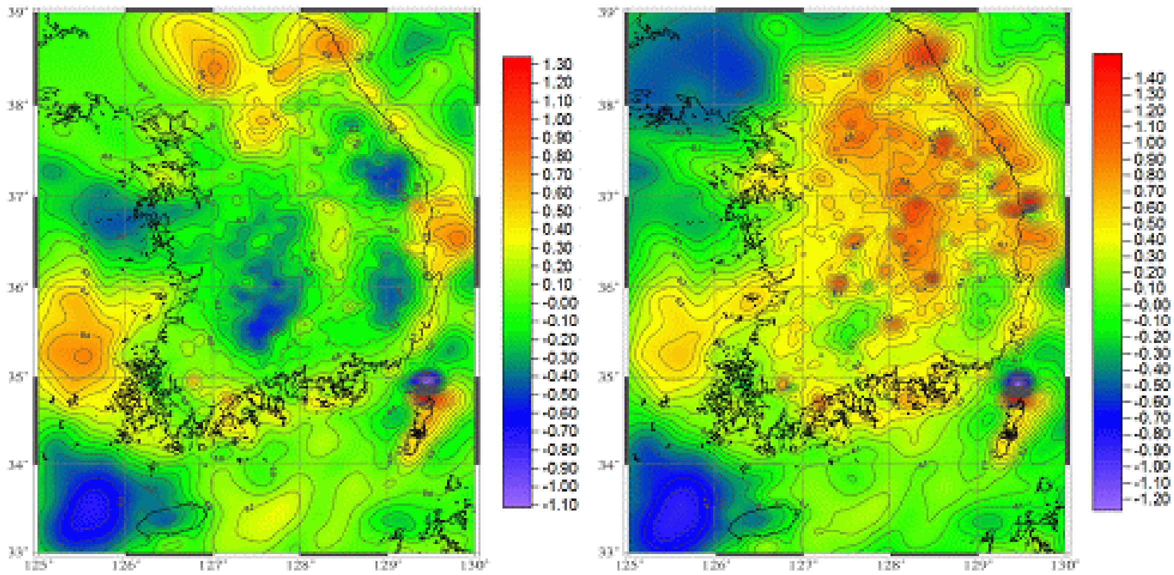
which is to prevent the influence of the medium wavelength element (or error) due to residual gravity anomaly from flowing into the long wavelength element obtained through spherical harmonic expansion of earth's gravity field model. Accordingly, in order to determine the final residual geoid height, we used total 6,296 residual gravity anomalies from which the direct effect of the terrain on the gravity measurement was removed by applying the reference gravity anomaly of EGM2008 model expanded in spherical harmonics to the maximum degree of 2160 and RTM conversion method. The final residual geoid height was modeled in grid interval of latitude 0.0125˚ and longitude 0.01667˚ (about 1.4 km x 1.5 km) for the whole area of Korea (latitude 33˚N ~ 39˚N and longitude 125˚E ~ 130˚E). For determination of precise residual geoid height, multi-band spherical FFT was conducted for application of Molodensky formula to a sphere, and integration cap size 0.2˚ and Wong-Gore modified kernel of maximum 150 degrees were applied in order to minimize the influence of the constant error and truncation error included in the residual gravity anomaly during Stokes integration on the long wavelength element.

3. RESIDUAL GEOID MODELING USING LSC METHOD

Residual Geoid Height Modeling using LSC (Least Square Collocation) is a method designed by Moritz (1980), which uses a mathematical method to determine the shape of earth or gravity field by applying variance of observation values calculated using auto-correlation of gravity observation values to least square method. The core concept of this method is to grasp the mutual relation between gravitational signal and noise given by gravity observation data and, using the same, to predict gravitational signal at the points where observations have not been made. LSC method has an advantage that the modeling error of residual geoid can be directly checked when it is compared with classic residual geoid height modeling methods (Stokes method, FFT method, etc.) and is known to be applicable to diverse areas such as detection of gross errors included in observation data and determination of gravity data distribution density for improvement of geoid accuracy (Tscherning, 1994).
The basic assumption made to determine residual geoid using LSC method is that the potential anomaly (T) is a harmonic function which satisfies Lapalce equation, according to which the potential anomaly at a point on the surface of the earth (P) should be identical with the potential anomaly for a specific earth's gravity field model (EGM2008, etc.) at the same point (Q). However, P and Q actually come to be on a same normal line based on the mass and the density of the terrain and indicate the difference in potential anomaly which is expressed in a linear relation with residual gravity anomaly. The method to consider such potential anomaly as an error and to do distribution in a way such an error is minimized in space, that is to say, to determine the plane on which the mean square error is minimized is the very overall concept of determination of residual geoid height using LSC method. But, the plane determined like this should satisfy the function expressed in spherical harmonic and, in order to satisfy this and to minimize the mean square error at the same time, the degree of covariance between the reference gravity anomaly and residual gravity anomaly should be clearly determined.
When R-R technique of LSC method is applied, the degree-variance of gravity anomaly changes in accordance with the maximum degree N used for spherical harmonic expansion of gravity field model, and, if the gravity observation data and the gravity field model used for LSC method do not well conform with each other as gravity data for the object region has not been included in the data used to develop the gravity field model, it comes to change very irregularly. This is because the first degree of degree-variance of the gravity anomaly is very closely related with the error of the gravity field model coefficient. Accordingly, we can assume that degree-variance σ_n has a proportional relationship with error-degree-variance σ_n^E at the minimum (Tscherning, 1994). Eventually, though we know that, if degree-variance of gravity anomaly can be determined, degree-variance of the potential can be also determined through the proportional relationship, in order to determine covariance function of gravity anomaly, infinite determinations for numerous degree-variance values should be made. In order to resolve such a problem, a degree-variance model comprised of functional relation between degree and degree-variance is used (Tscherning, 1994). Though there are diverse degree-variance models, Tscherning/Rapp model proposed by Tscherning et al. (1974) was used in this study in order to determine degree-variance of gravity anomaly. Finally, the parameters of degree-variance function for residual geoid height modeling were determined to be α = 0.420, R_B = 6,375.521 km, and A = 564,526 (m/s^2) respectively, and also C_0 = 118.40 mGal^2 and ψ_1 = 0.126° (about 13 km) respectively.

4. RESULTS AND DISCUSSIONS

In this section, in order to more clearly judge the adequacy of residual geoid height modeling using LSC method, we compared the residual geoid height of the whole area in Korea modeled by 4-band spherical FFT method and the residual geoid height modeled by LSC method. For this, a statistical analysis was carried out for the difference between the residual geoid height modeled by each method.



(a) FFT method (b) LSC method

Figure 2. Distribution of residual geoid height from each modelling method (Unit: m)

Figure 2 shows at the same time the residual geoid heights modeled by FFT method and LSC method. As we can see from the figure, though the residual geoid height distribution shows a similar appearance excluding a part of North Korea, we can see that distribution in a whole central region (mountainous region) shows a considerably different appearance. In order to analyze the difference in distribution of residual geoid heights, the distribution density was analyzed by superimposing gravity data onto the residual geoid height, and the difference of each developed gravimetric geoid from geometric geoid was preferentially analyzed.
As a result of analysis, we could see that the accuracy of gravimetric geoid developed by FFT method was about ±0.123 m producing a result better than the accuracy of the gravimetric geoid developed by LSC by about 10 cm. Through this result, we presume that it is appropriate to apply FFT method to development of precise geoid model of Korea. In fact, as same long wavelength and short wave length elements were used when calculating gravimetric geoid, such difference in accuracy can be presumed to have reflected the difference in residual geoid heights as it is. Eventually, it means that the accuracy of the residual geoid obtained by LSC method is relatively lower than that obtained by FFT method. The reasons for such a result can be largely divided into two. The first is presumed to be insufficiency of distribution density of observation data and the second is because the observation data cannot maintain homogeneity with other observation data as the error included in the observation data is too big. That is to say, the covariance and correlation distance of the residual gravity anomaly determined when applying LSC method were determined using the whole data and the reason is that an accurate LSC method may not have been applied as the determined correlation distance is out of the distribution density of the data within the anomaly region or the determined correlation distance has a very week correlation with the covariance of the whole determined region due to the error included in the observation data.

5. CONCLUSIONS

In this study, we intended to present a more efficient geoid determination method by applying residual geoid modeling using FFT method and LSC method respectively in the context of application of R-R (Remove and Restore) technique as a general technique for gravimetric geoid model and by analyzing their accuracy.
As a result, in the case of LSC method, an accurate residual geoid height model could not be applied due to the error in gravity observation data, insufficiency in the distribution density and the influence of characteristics of each region on analysis of covariance, and, in the case of multi-band spherical FFT, we could achieve an accurate residual geoid height modeling which minimizes the influence of the spherical surface error, truncation error and the error in the gravity observation data through application of proper integration cap size and modified kernel for Stokes integration. When performing multi-band spherical FFT, 4 multi-bands, integration

cap size 0.2° and Wong-Gore modified kernel of maximum degree 150 were applied together with 100% zero padding, through which a residual geoid height with an accuracy improved by about 10 cm in comparison to that of LSC method was calculated.

In the case of residual geoid modeling using FFT method, we presume to have effectively prevented the problem which occurs due to the error and declination included in the gravity observation data and the distribution region of the gravity observation data by using optimum integration cap size and modified Stokes kernel, and FFT method was evaluated to be a method more stable than LSC method in modeling residual geoid height in Korean region. Accordingly, we think it is somewhat impractical to apply LSC method for determination of precise geoid of Korea at present, and, in order to apply LSC method in the future, we believe that gravity observation data which has a high distribution density and homogeneity for the whole Korean region should be secured and determination of proper covariance model should be preceded.

REFERENCES

Lee, D.H. (2008) Development of Precise Hybrid Geoid Model in Korea, Ph.D. Dissertation, Sungkyunkwan University.

Forsberg, R. and Featherstone, W.E. (1998) Geoids and cap-sizes. Geodesy on the Move: Gravity, Geoids, Geodynamics and Antarctica, Forsberg, R., Feissl, M. and Dietrich, R. eds., Springer, Berlin, Germany, pp. 194-200.

Moritz, H. (1980) Advanced Physical Geodesy. H. Wichmann Verlag, Karlsruhe.

Stokes, G.G. (1849) On the variation of gravity at the surface of the Earth. Transactions of the Cambridge Philosophical Society, Vol. 8, pp. 672-695.

Tscherning, C.C. (1994) Local approximation of the gravity potential by least-squares collocation. Proceedings of the international summer school on local gravity field approximation, Beijing, China, August 21 - September 4.

Wong, L. and Gore, R. (1969) Accuracy of geoid heights from modified Stokes kernels. Geophysical Journal of the Royal Astronomical Society, Vol. 18, pp. 81-91.

Yun, H.S. (1999) Precision geoid determination by spherical FFT in and around the Korean peninsula, Earth Planets and Space, Vol. 51, pp. 13-18.