# PPP-RTK PLATFORM PERFORMANCE BASED ON SINGLE-FREQUENCY GPS DATA


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**ABSTRACT:** As an improvement over 'conventional' PPP, Real-Time Kinematic Precise Point Positioning (PPP-RTK) is a promising technique for high-precision (cm-level) carrier-phase based remote sensing platform positioning. The key to obtain these very precise positions is that the user should be able to resolve the ambiguities in the phase data to their integer values, as then his phase data starts to act as if they were very precise code data. In order to do so, the user needs to apply corrections to his GPS data. In addition to corrections for the satellite clocks and ionospheric delays, as with 'conventional' PPP, crucial to restore the integerness of the ambiguities is that the PPP-RTK user needs appropriate corrections for the satellite phase hardware biases. In our approach these corrections are determined by a regional network of CORS stations. To provide the most precise corrections to the PPP-RTK user, the corrections should be based on a solution in which the network ambiguities are resolved to their integer values. Furthermore, in our approach the ambiguity-fixed network ionospheric delays are interpolated to the approximate user location. So far very fast -even instantaneous- PPP-RTK integer ambiguity resolution performance has been reported based on dual-frequency GPS data. However, the technique becomes more attractive when it can be applied to users that operate with low-cost mass-market single-frequency receivers as well. In this contribution we demonstrate that our PPP-RTK approach can be applied to these users without any modification. Results are presented of the performance of single-frequency PPP-RTK for both a high-grade and a low-grade GPS receiver. The conclusion reads that single-frequency PPP-RTK integer ambiguity resolution is feasible, even using a low-cost receiver: following an initialization time of less than 5 minutes the correct integers can be resolved in real-time, thus providing cm-level positioning.


## 1. INTRODUCTION

The technique of Precise Point Positioning (PPP) is based on GPS carrier phase and code (pseudo-range) observations of a single receiver, employing corrections for, among others, satellite orbits, clocks and ionospheric delays obtained from a worldwide network of GPS stations, for example the permanent GPS network of the International GNSS Service (IGS). PPP was introduced by Zumberge et al. (1997) and the attainable instantaneous precision for a single-frequency user who employs global corrections is typically at the level of a few dm (Bree and Tiberius, 2011).

The key to fast and cm-level PPP lies in resolving the ambiguities that are present in the phase data to integer values. Unfortunately, with the standard PPP technique this is not possible, because the ambiguities cannot be separated from the satellite hardware biases in the phase and code data. In this contribution we will present an approach that allows the single-receiver user to perform integer ambiguity resolution within short time spans and consequently enable high-precision positining. This 'PPP-RTK' approach is like standard PPP based on applying corrections for satellite clocks and ionospheric delays, but crucial to enable ambiguity resolution are corrections for satellite phase biases, and these should have an appropriate definition. These corrections should be estimated simultaneously from a CORS network and transmitted to users. Advantage of our PPP-RTK approach is that it is not only suitable for dual-frequency users, but also for users employing low-cost single-frequency receivers. Although dual-frequency users can do without ionospheric corrections, these are essential to speed up integer ambiguity resolution for single-frequency users; without ionospheric corrections single-frequency PPP-RTK would suffer from very long convergence times. Moreover, it is doubtful whether ambiguity resolution would still make sense, since the float precision would already be good enough after a sufficiently long time span.

In this paper the PPP-RTK concept is demonstrated based on corrections determined from a regional CORS network with inter-station distances of less than 100 km. Advantage of such a relatively dense regional network is that the ionospheric corrections can be determined much more precise than using a global network and this should benefit single-frequency applications. It is assumed that the regional network provides corrections for satellite clocks, phase biases and ionospheric delays, but not for satellite orbits. These should be computed by the user by employing the IGS orbit information. The paper is set up as follows. Section 2 reviews the GPS phase and code observation equations, while Section 3 discusses the CORS network corrections that enable both PPP and PPP-RTK. Results of the performance of both techniques based on single-frequency GPS data collected with both a high-grade and a low-cost receiver are given in Section 4. A special focus in this section is on the possible temporal stability of the satellite phase biases. Section 5 ends the paper with conclusions.

## 2. BETWEEN-SATELLITE DIFFERENCED GPS PHASE AND CODE OBSERVATION EQUATIONS

Let us assume a receiver *r* tracking multi-frequency GPS phase and code data. Since the focus of this paper is on the satellite-dependent effects, our starting point is formed by the *between-satellite differences* (SD) of the linearized phase and code observation equations from which all receiver-specific unknowns are removed, in units of distance:

$$\begin{aligned} E(\phi_{r,j}^{ps}) &= -(u_r^{ps})^T x_r - (dt^{ps} + \delta_{,j}^{ps}) - \mu_j \iota_r^{ps} + \psi^{ps}\tau_r + \lambda_j M_{r,j}^{ps} \\ E(p_{r,j}^{ps}) &= -(u_r^{ps})^T x_r - (dt^{ps} + d_{,j}^{ps}) + \mu_j \iota_r^{ps} + \psi^{ps}\tau_r \end{aligned} \tag{1}$$

where the differences are formed between satellite *s* and a chosen pivot satellite, denoted using the superscript *p*: $(\cdot)^{ps} = (\cdot)^s - (\cdot)^p$. It is assumed that the positions of the satellites are known. In these observation equations $E(\cdot)$ denotes the mathematical expectation, $\phi_{r,j}^{ps}$ and $p_{r,j}^{ps}$ the observed-minus-computed SD observables for phase and code respectively on frequency *j*, $u_r^{ps}$ the SD receiver-satellite line-of-sight vector, $x_r$ the (incremental) receiver coordinates, $dt^{ps}$ the SD satellite clock error, $\delta_{,j}^{ps}$ and $d_{,j}^{ps}$ the SD satellite phase and code hardware biases, $\iota_r^{ps}$ the SD ionospheric delay with $\mu_j = (\lambda_j^2 / \lambda_1^2)$ its frequency-dependent coefficient, $\tau_r$ the zenith tropospheric delay (ZTD), with $\psi^{ps}$ the SD mapping function, $\lambda_j$ the wavelength, and $M_{r,j}^{ps} = -\varphi_{,j}^{ps}(t_0) + N_{r,j}^{ps}$ the SD phase ambiguity that consists of the initial phases of satellite $\varphi_{,j}^{ps}(t_0)$, plus an integer SD ambiguity $N_{r,j}^{ps}$, both in units of cycle. All clock errors and hardware biases are given in units of distance. Both phase and code data are assumed to be a priori corrected for effects such as hydrostatic troposphere, phase center offsets, phase wind-up, solid earth tides, ocean loading, etc. More details on these corrections can be found in (Kouba and Héroux, 2001).

## 3. CORS-BASED SINGLE-FREQUENCY PPP AND PPP-RTK

In this section the corrections are discussed that need to be estimated from a GPS CORS network in order for a user to carry out PPP as well as PPP-RTK. For PPP-RTK it is shown that using satellite phase bias corrections with appropriate definition the ambiguities for the user become integer.

### 3.1 Regional CORS Network Corrections

The regional CORS GPS network processing is based on keeping the positions fixed of both receivers (since these are precisely known), as well as of the satellites (from IGS orbit information). Unknown network parameters are then, in terms of between-satellite single differences: satellite clocks, satellite phase biases, ZTDs, ionospheric delays and phase ambiguities. Since the network model is not of full rank, the network parameters are only estimable as combinations of parameters in order to remove the network's rank deficiency. Here we will show our current choice for the combination of these network parameters.

The estimable satellite clock parameters from the regional network can be shown to be a function of the true satellite clocks, plus the code biases on L1 and L2, and the ZTD of the network's pivot receiver (Odijk et al., 2011):

$$\boxed{\widetilde{dt}_{,IF}^{ps} = (dt^{ps} + \tfrac{\mu_2}{\mu_2-\mu_1} d_{,1}^{ps} - \tfrac{\mu_1}{\mu_2-\mu_1} d_{,2}^{ps}) - \psi^{ps}\tau_1 = dt^{ps} + d_{,1}^{ps} + \tfrac{\mu_1}{\mu_2-\mu_1} DCB^{ps} - \psi^{ps}\tau_1} \tag{2}$$

It is remarked that the terms between brackets correspond to the ionosphere-free satellite clocks as provided by the IGS that are applied in 'standard PPP', i.e. $dt_{,IF}^{ps} = dt^{ps} + \frac{\mu_2}{\mu_2-\mu_1} d_{,1}^{ps} - \frac{\mu_1}{\mu_2-\mu_1} d_{,2}^{ps}$. Our 'regional-based' satellite clock product is however biased by the ZTD of the network's pivot receiver, since for regional CORS networks ZTDs are not estimated for each receiver, but relative to the pivot receiver of the network. Furthermore, the so-called

Differential Code Bias (DCB) can be recognized, which is defined as $DCB^{ps} = d_{,1}^{ps} - d_{,2}^{ps}$ (Schaer, 1999). Essential to the performance of single-frequency users is furthermore that the network should provide the user with ionospheric corrections. For 'standard PPP' these can be obtained from a Global Ionospheric Map, but for a dense regional CORS network a more sophisticated ionospheric product can be generated: the ionospheric delay interpolated at the approximate user location from the network ionospheric delays (Odijk et al., 2011):

$$\boxed{\tilde{\iota}_r^{ps} = \iota_r^{ps} + \tfrac{1}{\mu_2-\mu_1} DCB^{ps}} \tag{3}$$

with $\iota_r^{ps}$ the ionospheric delay interpolated from the network ionospheric estimates. Similar to the satellite clock parameter, in the estimable ionospheric parameter the DCB shows up. To enable integer ambiguity resolution for the PPP user the CORS network should provide corrections for the satellite phase bias parameters, defined as follows (Odijk et al., 2011):

$$\boxed{\tilde{\delta}_{,1}^{ps} = -\lambda_1 \left[ M_{1,1}^{ps} - \tfrac{1}{\lambda_1}(\delta_{,1}^{ps} - d_{,1}^{ps} - \tfrac{2\mu_1}{\mu_2-\mu_1} DCB^{ps}) \right]} \tag{4}$$

The between-satellite phase bias parameter is in fact a combination of the true between-satellite phase bias $\delta_{,1}^{ps}$, biased by a combination of the satellite code biases on L1 and L2 (through $d_{,1}^{ps}$ and $DCB^{ps}$), plus the (non-integer) ambiguity of the network's pivot receiver ($M_{1,1}^{ps}$). We finally emphasize that the network parameters should be transmitted to the users, after the network ambiguities are fixed to integers, such that the user has the disposal of corrections with the best possible precision.

### 3.2 PPP Based on Regional Network Corrections

With the network corrections identified, the single-frequency observation equations for 'regional PPP' can be given as:

$$\begin{aligned} E(\phi_{r,1}^{ps} + \Delta\tilde{\phi}_{r,1}^{ps}) &= -(u_r^{ps})^T x_r + \psi^{ps}\tilde{\tau}_r + \lambda_1 \tilde{M}_{r,1}^{ps} \\ E(p_{r,1}^{ps} + \Delta\tilde{p}_{r,1}^{ps}) &= -(u_r^{ps})^T x_r + \psi^{ps}\tilde{\tau}_r \end{aligned} \tag{5}$$

with the phase and code observables now corrected for the ionosphere-free satellite clocks and interpolated ionospheric delays from the CORS network, see Eqs. (2) and (3):

$$\boxed{\begin{aligned} \Delta\tilde{\phi}_{r,1}^{ps} &= \widetilde{dt}_{,IF}^{ps} + \mu_1 \tilde{\iota}_r^{ps} \\ \Delta\tilde{p}_{r,1}^{ps} &= \widetilde{dt}_{,IF}^{ps} - \mu_1 \tilde{\iota}_r^{ps} \end{aligned}} \tag{6}$$

The estimable ZTD for the PPP user is relative to the network's pivot station: $\tilde{\tau}_r = \tau_r - \tau_1$. Consequence of correcting the phase data in this way is that the estimable ambiguity term becomes:

$$\tilde{M}_{r,1}^{ps} = M_{r,1}^{ps} - \tfrac{1}{\lambda_1}(\delta_{,1}^{ps} - d_{,1}^{ps} - \tfrac{2\mu_1}{\mu_2-\mu_1} DCB^{ps}) \tag{7}$$

From this equation it can be clearly seen that the estimable ambiguity parameter is *not* an integer.

### 3.3 PPP-RTK Based on Regional Network Corrections

We will now show that integer ambiguity resolution for the PPP user becomes possible when correcting his phase data using the network's satellite phase biases. The observation equations remain exactly the same as in Eq. (5), as well as the corrected code observables, however the correction to be applied to the phase observable becomes:

$$\boxed{\begin{aligned} \Delta\tilde{\phi}_{r,1}^{ps} &= \widetilde{dt}_{,IF}^{ps} + \tilde{\delta}_{,1}^{ps} + \mu_1 \tilde{\iota}_r^{ps} \\ \Delta\tilde{p}_{r,1}^{ps} &= \widetilde{dt}_{,IF}^{ps} - \mu_1 \tilde{\iota}_r^{ps} \end{aligned}} \tag{8}$$

As consequence, the satellite phase bias correction $\tilde{\delta}_{,1}^{ps}$ will eliminate the phase-code bias term $(\delta_{,1}^{ps} - d_{,1}^{ps} - \tfrac{2\mu_1}{\mu_2-\mu_1} DCB^{ps})$ in the biased ambiguity term, see Eq. (7), such that the estimable phase ambiguity parameter becomes a combination of the between-satellite ambiguities of the PPP receiver and the between-satellite ambiguities of the network's pivot receiver, i.e.:

$$\tilde{M}_{r,1}^{ps} = M_{r,1}^{ps} - M_{1,1}^{ps} = N_{r,1}^{ps} - N_{1,1}^{ps} = N_{1r,1}^{ps} \tag{9}$$

which is a *double-differenced* ambiguity and thus integer. The estimable PPP ambiguity has become a double difference because of the ambiguity information of the network's pivot receiver 'hidden' in the satellite phase bias correction.

High-precision PPP based on integer ambiguity resolution can now be realized using the following stepwise approach: (i) *Float solution:* the user solves the model (5) applying the PPP-RTK corrections as given in Eq. (8) using standard least-squares; the position solution then corresponds to a 'regional PPP' solution (ii) *Integer ambiguity* *resolution:* the float ambiguity solution is input to the LAMBDA method (Teunissen, 1995) to resolve the double-differenced integers; (iii) *Fixed solution:* If the integer solution can be accepted, a 'regional PPP-RTK' solution is computed based on model (5) with the ambiguities held fixed.

## 4. RESULTS OF CORS-BASED PPP AND PPP-RTK

In order to test the performance of our PPP-RTK concept we determined corrections from a regional CORS network and applied these to single-frequency user data. The CORS network is depicted in Figure 1 and consists of four stations of the GPS Network Perth in Western Australia, a privately operated CORS network. The four stations are at distances of about 60 km and are all equipped with the same Trimble NetR5 receivers. The location at Curtin University Bentley campus (CUT0) was assigned as (static) rover station. At this station, GPS data were collected using two receivers: a *high-grade* dual-frequency Trimble NetR9 receiver, plus a *low-grade* single-frequency u-blox AEK-4T receiver. For all CORS network stations dual-frequency phase and code observations have been collected above a cut-off elevation of 10 deg during the full day of 23 October 2010 with a sampling interval of 30 sec.

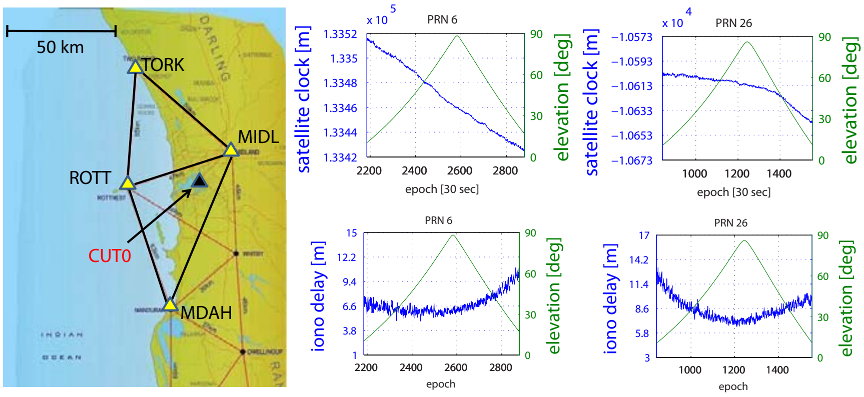


**Figure 1:** *(Left)* CORS network (yellow triangles) used for generating the PPP(-RTK) products for user location CUT0 (black triangle). *(Right)* Examples of ambiguity-fixed satellite clock and interpolated ionospheric delay estimates for full arcs of PRNs 6 and 26 during 23 October 2010, where an epoch interval is 30 sec is used. In the graphs the satellite's elevation is plotted as well.

### 4.1 Results of Regional CORS-Based Network Corrections

From the dual-frequency CORS network data precise estimates for satellite clock parameters, satellite phase bias parameters and interpolated ionospheric delays (for location CUT0) were obtained after network ambiguity resolution. Examples of estimates of the ambiguity-fixed satellite clocks and interpolated ionospheric delays are shown in Figure 1 for full arcs of PRN 6 and PRN 26. It can be seen that both satellite clocks and ionospheric delays are changing significantly in time. The temporal behavior of the satellite phase biases, the parameters that enable PPP-RTK, is however much different. Figure 2 (left 4 columns) depicts the ambiguity-fixed L1 satellite phase bias estimates for the full day and for all satellites in view by the network. For almost all satellites more than one graph is included, because during a full day the same satellite can be tracked more than once. It can be seen that the phase bias estimates seem to be quite stable during the arc: the visible fluctuation is due to the noise in the estimates (several dm), but the moving average (depicted as the yellow curve in each of the graphs) only shows little fluctuation. If the satellite phase biases turn out to be sufficiently stable, they can be transmitted to users with a less frequent rate than at every epoch.

This temporal stability of the satellite phase bias parameters has been further analyzed by means of statistical hypothesis testing. Per satellite arc a null hypothesis ($H_0 : E(\tilde{\delta}(i)) = \text{constant}\ \forall\, i$, i.e. the satellite phase bias parameter is time-constant) is tested against an alternative hypothesis ($H_A : E(\tilde{\delta}(i)) \neq \text{constant}\ \forall\, i$, i.e. the satellite phase bias parameter is not time-constant). It can be shown that the test statistic corresponds to the overall model test (Teunissen, 2000) of the model having as observations the satellite phase bias time series per arc and as unknown parameter the constant phase bias, incorporating the variance matrices of the time-varying phase biases obtained from the network processing in the stochastic model. This overall model test is not only executed based on all phase biases for one arc, but starting from the second epoch of the arc (note that at the first epoch there is no redundancy) as to demonstrate the effect of accumulating the temporal phase biases on the test outcomes. The overall model test statistic has a central F-distribution with the first set of degrees of freedom equal to the accumulated number of epochs minus 1 (the redundancy) and the second set of degrees of freedom set to infinity. The right four columns of Figure 2 show the overall model test outcomes (blue curve), including critical values. In order to see how sensitive the critical value is to the choice of significance level, the critical value is computed based on a significance level of 0.01 (red curve), as well as for a significance level of 0.05 (green curve). From the graphs it can be seen that for none of the satellite arcs the test outcomes exceed both critical values. From this we may conclude that based on this dataset there is no reason to assume that the phase bias parameters are unstable in time during the arc of a satellite.

**Figure 2:** *(Left)* Ambiguity-fixed L1 satellite phase bias parameters per satellite arc for the full day of 23 October 2010. The satellite phase biases (in blue) are expressed in meters and for each arc the moving average is plotted (in yellow). In each graph the satellite's elevation is plotted as well (in deg; green curve). *(Right)* Test outcomes (blue) vs. critical value based on significance level of 0.01 (red) and critical value based on significance level of 0.05 (green) to test the stability of the satellite phase biases.

### 4.2 Results of Regional CORS-Based PPP and PPP-RTK

In a next step the network corrections are applied to correct the user's high-grade and low-grade receiver data and perform PPP and PPP-RTK. First the satellite clock parameters plus interpolated ionospheric delays are used to enable 'regional-based' PPP. Figure 3 (left two columns) depicts the horizontal position scatter solutions and vertical time series obtained from processing of the observation model (6). The model is solved in a truly epoch-by-epoch manner, where the position and ambiguities are solved for each epoch, irrespective of the solutions of other epochs. The North-East-Up components are obtained by comparing the position outcomes with known (ground-truth) coordinates of station CUT0. While the (empirical) horizontal precision of PPP with the high-grade receiver is at the

3 dm level and at sub-m level for the vertical component, the results for the low-grade receiver are about a factor 2 worse. This difference is due to the quality of the code data, which is a factor 2 worse for the ublox receiver compared to the Trimble receiver. The epoch-by-epoch PPP solutions are known to be fully driven by these code data.

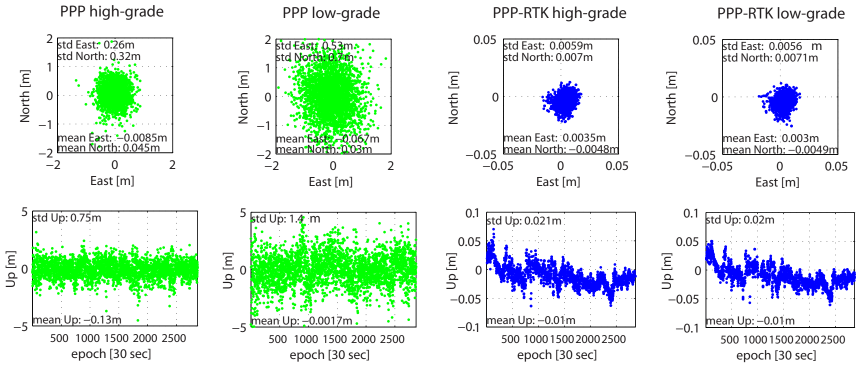


**Figure 3:** PPP and PPP-RTK positioning results based on single-frequency GPS phase and code data. It is remarked that the PPP results are obtained by epoch-by-epoch kinematic processing, while those for PPP-RTK are based on accumulation of epochs.

For the same high-grade and low-grade single-receiver data PPP-RTK was subsequently enabled by correcting the phase data for the satellite phase biases. For these results these have been transmitted to the user at every epoch. Due to the weakness of the single-frequency observation model (6), instantaneous or epoch-by-epoch *ambiguity* resolution was not feasible; however ambiguity resolution was successful by accumulating epochs (based on the constancy of the ambiguities); for the Trimble receiver after 2.5 min on average and after 4 min on average for the ublox receiver. After ambiguity resolution, the horizontal precision of the kinematically solved fixed positions is at sub-cm level, while the vertical precision is about 2 cm, for both types of receivers; see Figure 3 (right two columns).

## 5. CONCLUSIONS

In this paper it has been shown that integer ambiguity resolution is possible for single-receiver Precise Point Positioning of platforms. The observation model of this 'PPP-RTK' method is intrinsically the same as for 'standard PPP'; the difference lies in the corrections applied to the user's phase data. One can do standard PPP with corrections for among others satellite clock and ionospheric delay estimated from a CORS network; however the essential correction that enables integer ambiguity resolution is the satellite phase bias parameter, which is in fact a combination of the (true) satellite phase bias, together with the (non-integer) ambiguity of the network's pivot station and dual-frequency satellite code biases. Our data processing indicates that these satellite phase bias parameters seem to be relatively stable during a satellite arc. Our PPP-RTK approach is not only suitable for dual-frequency but also for single-frequency applications. Results demonstrate that single-frequency PPP ambiguity resolution is feasible, based on convergence times of less than 5 minutes, even using low-grade receivers. The kinematic position accuracy with the integer ambiguities fixed is at sub-cm level horizontally, and at the level of a few cm in the vertical direction.

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