GNSS-BASED ATTITUDE DETERMINATION FOR REMOTE SENSING PLATFORMS

N Nadarajah*[1], PJG Teunissen[2], and D Odijk[3]

*[1]Research Fellow, [2]Professor, [3]Research Fellow,
GNSS Research Centre, Department of Spatial Sciences,
Curtin University, GPO Box U1987, Perth, WA 6845, Australia; Tel: +61 8 92663157;
E-mail: n.nadarajah@curtin.edu.au, p.teunissen@curtin.edu.au, and d.odijk@curtin.edu.au

[2]Professor, Delft Institute of Earth Observation and Space Systems (DEOS),
Delft University of Technology, PO Box 5058, 2600 GB Delft, The Netherlands.

KEY WORDS: GNSS, Attitude Determination, Constrained Integer Least-Squares, MC-LAMBDA, Carrier-Phase Ambiguity Resolution.

ABSTRACT: Global Navigation Satellite Systems (GNSS)-based attitude determination is a valuable technique for the estimation of platform orientation. Precise attitude determination using multiple GNSS receivers/antennas mounted on a remote sensing platform relies on successful resolution of the carrier phase integer ambiguities. The LAMBDA method has proven to be an efficient method to solve integer least squares problems. This method is, however, only applicable to unconstrained and/or linearly constrained models, but not to quadratically constrained models such as the GNSS attitude model. For a set of GNSS antennas rigidly mounted on a platform, a number of nonlinear geometrical constraints can be exploited for the purpose of strengthening the underlying observation model and subsequently improving the capacity of fixing the correct set of integer ambiguities. In this contribution, we describe and test the Multivariate Constrained (MC-) LAMBDA method, which effectively makes use of the known antenna geometry. Reliable and instantaneous integer estimation is particularly a challenge for single-frequency applications with low cost GNSS receivers. With our field tests, we show the potential of stand-alone, unaided, single-frequency, single epoch attitude determination.

1 INTRODUCTION

Precise attitude determination is a prerequisite for remote sensing applications. For instance, estimating pointing directions for remote sensors such as radars and laser scanners requires the knowledge of platform orientation. Multiple GNSS receivers/antennas mounted rigidly on the platform can be used to determine platform orientation, see e.g., (Cohen, 1992; Crassidis and Markley, 1997; Psiaki, 2006). GNSS-based attitude determination system offers several advantages including that it is driftless and requires less maintenance.

GNSS-based attitude determination requires a precise relative positioning solution, that can be provided in principle by the very precise GNSS carrier phase observables. The phase observables are however biased by unknown integer ambiguities, that must be resolved in order to fully exploit their higher precision. Carrier phase integer ambiguity resolution is therefore the key to high-precision GNSS positioning. The Least squares AMBiguity Decorrelation Adjustment (LAMBDA) method (Teunissen, 1995) is currently the standard method for solving unconstrained GNSS ambiguity resolution problems, see, e.g., (Boon and Ambrosius, 1997; Huang et al., 2009). For unconstrained and linearly constrained GNSS models, the method is known to be optimal in the sense that it provides integer ambiguity solutions with the highest possible success-rate and in a numerically efficient way (Teunissen, 1999; Verhagen and Teunissen, 2006).

In this contribution we focus on the problem of fixing the correct integer ambiguities for data collected on a frame of antennas firmly mounted on a rigid platform: the relative positions between the antennas are assumed to be known and constant. In such configurations, the baselines lengths and the angles between them are known, resulting in a set of nonlinear constraints posed on the baseline vectors which can be exploited to strengthen the underlying observation model (attitude model). To exploit these constraints, we make use of the Multivariate Constrained (MC-) LAMBDA method (Teunissen, 2007; Giorgi et al., 2010).

In this contribution, we illustrate the principles of the MC-LAMBDA method and we show its performance by means of a static test with low-cost (U-Blox) receivers and with a kinematic test using data from an aircraft experiment. The most challenging application, being single-frequency, single epoch GPS-only ambiguity resolution and attitude determination, is considered. The single-frequency case is of interest for many aerospace applications, where limits on weight and power consumption must often be respected (e.g., UAVs).

2 The GNSS-BASED ATTITUDE DETERMINATION

Let us consider a set of $r + 1$ antennas simultaneously tracking the same $s + 1$ GNSS satellites on a single frequency. The set of linearized Double Difference (DD) GNSS phase and code observations obtained on the r

baselines can be cast into a *multivariate* Gauss-Markov model as follows:

$$\begin{aligned} \mathrm{E}(Y) &= AZ + GB \qquad Z \in \mathbb{Z}^{s\times r}, B \in \mathbb{R}^{3\times r} && (1)\\ \mathrm{D}(\mathrm{vec}(Y)) &= Q_{YY} = P \otimes Q_{yy} && (2) \end{aligned}$$

where $\mathrm{E}(\cdot)$ and $\mathrm{D}(\cdot)$ denote the expectation and dispersion operator, $\otimes$ denotes the Kronecker product, $Z = [z_1, \ldots, z_r]$ is the $s \times r$ matrix of r unknown DD integer ambiguity vectors z_i, $B = [b_1, \ldots, b_r]$ the $3 \times r$ matrix of r unknown baseline vectors b_i, G is the $2s \times 3$ geometry matrix that contains the unit line-of-sight vectors, A is the $2s \times s$ matrix that links the DD data to the integer ambiguities, and P and Q_{yy} are known matrices of order $r \times r$ and $2s \times 2s$, respectively. Here, $\mathrm{vec}(\cdot)$ denotes the vec-operator, which transforms a matrix into a vector by stacking the columns of the matrix one underneath the other. Matrix P takes care of the correlation that follows from the fact that the r baselines have one antenna in common and matrix Q_{yy} takes care of the precision of the phase and code data.

Model (1) can be strengthened by making use of the a priori known body-frame antenna geometry. This allows us to reparametrize B as

$$B = RB_0 \tag{3}$$

with the unknown $3 \times q$ orthogonal matrix R ($R^T R = I_q$) and the known $q \times r$ matrix B_0 describing the known geometry of the antenna configuration in the body frame (q is the dimension of the span of the r baselines). Introducing this relation into model (1), gives the GNSS attitude model

$$\begin{aligned} \mathrm{E}(Y) &= AZ + GRB_0 \qquad Z \in \mathbb{Z}^{m\times n}, R \in \mathbb{O}^{3\times q} && (4)\\ \mathrm{D}(\mathrm{vec}(Y)) &= Q_{YY} = P \otimes Q_{yy} && (5) \end{aligned}$$

Our goal is to solve the above system in a least-squares sense, while taking the integer constraints on Z and the orthonormaility constraints on R into account. Hence, the minimization problem that will be solved reads

$$\min_{Z\in\mathbb{Z}^{s\times r}, R\in\mathbb{O}^{3\times q}} \|\mathrm{vec}\,(Y - AZ - GRB_0)\|^2_{Q_{YY}} \tag{6}$$

with $\|\cdot\|^2_Q = (\cdot)^T Q^{-1}(\cdot)$. The above problem does not admit a closed-form solution. In the following, we describe a three-step procedure for solving (6).

2.1 An Orthogonal Decomposition

Using an orthogonal decomposition of the objective function, problem (6) can be written as:

$$\begin{aligned} &\min_{Z\in\mathbb{Z}^{s\times r}, R\in\mathbb{O}^{3\times q}} \|\mathrm{vec}\,(Y - AZ - GRB_0)\|^2_{Q_{YY}} \\ &= \left\|\mathrm{vec}\left(\hat{E}\right)\right\|^2_{Q_{YY}} + \min_{Z\in\mathbb{Z}^{s\times r}} \left(\left\|\mathrm{vec}\left(\hat{Z} - Z\right)\right\|^2_{Q_{\hat{Z}\hat{Z}}} + \min_{R\in\mathbb{O}^{3\times q}} \left\|\mathrm{vec}\left(\hat{R}(Z) - R\right)\right\|^2_{Q_{\hat{R}(Z)\hat{R}(Z)}} \right) \end{aligned} \tag{7}$$

with $\hat{E}$ the matrix of least-squares residuals. For this decomposition we need $\hat{Z}$, $\hat{R}(Z)$ and their inverse-variance matrices. The so-called float solutions $\hat{Z}$ and $\hat{R}$, and their variance-covariance matrices, follow from

$$\begin{aligned} N \cdot \begin{bmatrix} \mathrm{vec}(\hat{Z}) \\ \mathrm{vec}(\hat{R}) \end{bmatrix} &= \begin{bmatrix} I_s \otimes A^T \\ B_0 \otimes G^T \end{bmatrix} Q^{-1}_{YY}\mathrm{vec}(Y) && (8)\\ N &= \begin{bmatrix} I_s \otimes A^T \\ B_0 \otimes G^T \end{bmatrix} Q^{-1}_{YY} \begin{bmatrix} I_s \otimes A & B_0 \otimes G \end{bmatrix} && (9) \end{aligned}$$

and

$$\begin{bmatrix} Q_{\hat{Z}\hat{Z}} & Q_{\hat{Z}\hat{R}} \\ Q_{\hat{R}\hat{Z}} & Q_{\hat{R}\hat{R}} \end{bmatrix} = N^{-1} \tag{10}$$

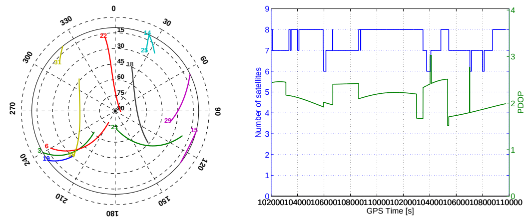
while the Z-constrained solution of R and its variance-covariance matrix are given as

$$\begin{aligned} \mathrm{vec}\left(\hat{R}(Z)\right) &= \mathrm{vec}(\hat{R}) - Q_{\hat{R}\hat{Z}} Q^{-1}_{\hat{Z}\hat{Z}} \mathrm{vec}\left(\hat{Z} - Z\right) && (11)\\ Q_{\hat{R}(Z)\hat{R}(Z)} &= Q_{\hat{R}\hat{R}} - Q_{\hat{R}\hat{Z}} Q^{-1}_{\hat{Z}\hat{Z}} Q_{\hat{Z}\hat{R}} && (12) \end{aligned}$$

2.2 The Multivariate Constrained LAMBDA method

Multivariate constrained minimization problem in (7) is equivalent to minimizing the cost function $C(Z)$:

$$\check{Z} = \arg \min_{Z\in\mathbb{Z}^{m\times n}} C(Z) \tag{13}$$



(a) Skyplot (b) Number of satellites and PDOP

Figure 1: Satellite visibility for 10° elevation cut-off

where
$$C(Z) = \left\| \mathrm{vec}(\hat{Z} - Z) \right\|^2_{Q_{\hat{Z}\hat{Z}}} + \left\| \mathrm{vec}\left(\hat{R}(Z) - \check{R}(Z)\right) \right\|^2_{Q_{\hat{R}(Z)\hat{R}(Z)}} \tag{14}$$

with
$$\check{R}(Z) = \arg \min_{R \in \mathbb{O}^{3\times q}} \left\| \mathrm{vec}\left(\hat{R}(Z) - R\right) \right\|^2_{Q_{\hat{R}(Z)\hat{R}(Z)}} \tag{15}$$

The cost function $C(Z)$ is the sum of two coupled terms: the first weighs the distance from the float ambiguity matrix $\hat{Z}$ to the nearest integer matrix Z in the metric of $Q_{\hat{Z}\hat{Z}}$, while the second weighs the distance from the conditional float solution $\hat{R}(Z)$ to the nearest rotation matrix R in the metric of $Q_{\hat{R}(Z)\hat{R}(Z)}$. This rigorous application of the orthonormal constraint results in non-ellipsoidal search space and requires the computation of a nonlinear constrained least-squares problem (15) for every integer matrix in the search space. In the MC-LAMBDA method, this problem is mitigated through the use of easy-to-evaluate bounding functions (Giorgi and Teunissen, 2010). Using these bounding functions, two efficient strategies, namely the *Search and Expansion* and the *Search and Shrink* strategies, were developed, see e.g. (Buist, 2007; Park and Teunissen, 2009; Giorgi et al., 2008; Giorgi and Buist, 2008). These techniques avoid the computation of (15) for every integer matrix in the search space, and compute the integer minimizer $\check{Z}$ efficiently.

To obtain the final attitude solution, $\check{Z}$ is substituted into (11), thus giving $\hat{R}(\check{Z})$. This solution has a much better accuracy than $\hat{R}$ (cf. 12), but it is, in general, still non-orthogonal. The sought-for orthogonal attitude solution is then finally obtained by solving (15) for $Z = \check{Z}$.

3 RESULTS

In this section the results of our experiments for testing the MC-LAMBDA method are presented. They are based on a static experiment using U-Blox receivers and a kinematic flight experiment using Septentrio receivers. The data of both experiments were processed in single-epoch, single-frequency mode with the standard LAMBDA method and with the MC-LAMBDA method.

3.1 A Static Test: U-Blox Experiment

In the static experiment, three U-Blox AEK-4T receivers were connected to three ANN-MS-0005 type antennas mounted on a symmetric frame. The experiment was conducted at Curtin University on 23 May 2011, for about two hours with sampling rate of 10 Hz (60000 epochs). Figure 1 shows satellite visibility (the sky-plot, the number of satellites, the PDOP values) during the experiment. We considered the following elevation dependent model (Euler and Goad, 1991) for the standard deviation of undifferenced observables

$$\sigma_\epsilon = \sigma_0 \left(1 + a_0 \exp\left(\frac{-\epsilon}{\epsilon_0}\right)\right) \tag{16}$$

where ϵ is the elevation angle of the satellite. The model parameters are $a_0 = 2.5$, $\epsilon_0 = 10°$, and $\sigma_0 = 1.3$ m (for code conservation) and 0.01 m (for phase observation).

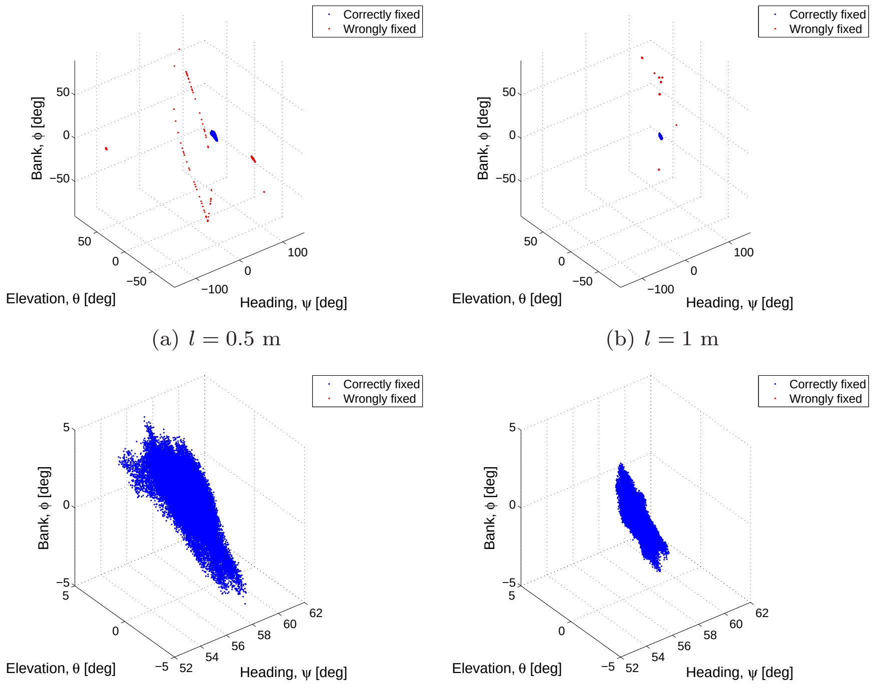
The planar antenna array geometry in the body-frame is given as

$$B_0 = \begin{bmatrix} l & 0 \\ 0 & l \end{bmatrix} \tag{17}$$

The following two cases have been considered: $l = 0.5$m and $l = 1$m. The antennas were rigidly fixed on a metal frame such that the both cases considered define the same body-frame. Table 1 reports the average computation time and the single-frequency, single-epoch success rates obtained by processing the U-Blox dataset with both the LAMBDA method and the MC-LAMBDA method. Even though the MC-LAMBDA method requires more computational effort than the LAMBDA method, it achieves higher success rates even for low-cost receiver data.

l	LAMBDA		MC-LAMBDA	
	Time [s]	Success rate [%]	Time [s]	Success rate [%]
$0.5m$	0.021	0.36	0.132	99.24
$1m$	0.016	0.77	0.112	99.05

Table 1: Average computation time [s] and integer resolution success rate [%]



(a) $l = 0.5$ m (b) $l = 1$ m

(c) $l = 0.5$ m, zoom (d) $l = 1$ m, zoom

Figure 2: The scatter plot of estimated attitude angles for the U-Blox experiment

Figure 2 shows the scatter plot of the estimated attitude angles for both baseline lengths. Table 2 summarizes the angular estimates (average values) and their precision (standard deviations) as function of baseline length. As the scatter plots and the table show, there are still some small systematic effects present in the solution. The dependence of angular accuracy on the baseline length can be seen clearly: the longer baseline results in higher precision. Both cases show a higher precision of the estimated heading angles, as horizontal positioning is more precise than vertical positioning using the GNSS satellites observed only from one side of the sky.

3.2 A Dynamic Test: Aircraft Attitude Estimation

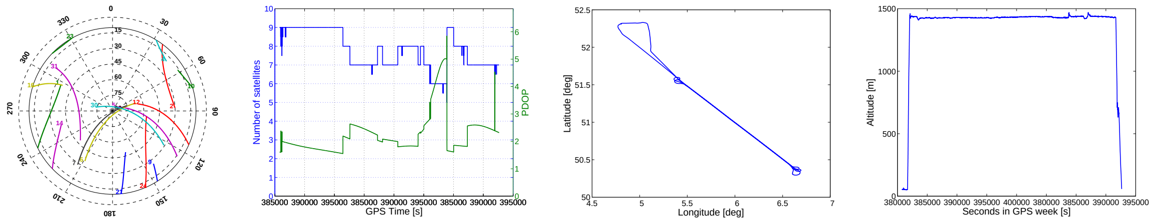
In this kinematic experiment, we used data collected on the Cessna Citation II aircraft of Delft University of Technology, The Netherlands. The aircraft equipped with three GNSS antennas: one on the body (reference antenna), approximately in the middle of the fuselage (S67-1575-96 type L1/L2 sensor system), one on the wing, and one on the nose (both L1 sensor system) forming the following antenna geometry,

$$B_0 = \begin{bmatrix} 4.90 & -0.39 \\ 0 & 7.60 \end{bmatrix} \quad \text{(m)} \tag{18}$$

All three antennas were connected to a Septentrio PolaRx2@ receiver, logging data for the entire duration of the flight, from 10:06 to 14:18 (UTC time). Figure 3 shows satellite visibility (the sky-plot, the number of satellites, the PDOP values) during the experiment. It also shows the ground track and the altitude profile of the flight calculated with the single-frequency observations collected on the reference antenna.

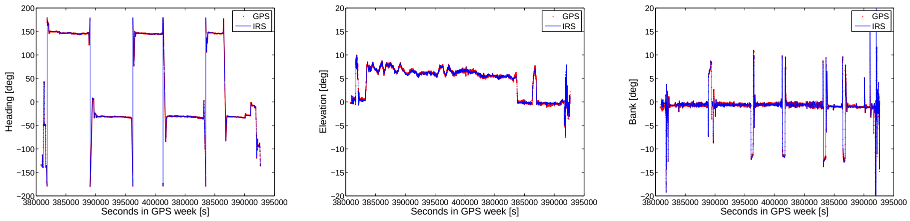
l [m]	Heading, ψ [deg]		Elevation, θ [deg]		Bank, ϕ [deg]	
	$\hat{\psi}$	$\sigma(\psi)$	$\hat{\theta}$	$\sigma(\theta)$	$\hat{\phi}$	$\sigma(\phi)$
0.5	56.83	0.54	-0.04	1.81	1.20	1.19
1	57.30	0.21	0.22	0.84	-0.21	0.82

Table 2: The three estimated attitude angles as function of baseline length l

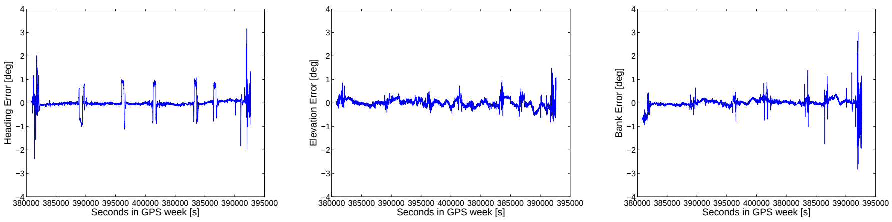


(a) The skyplot (b) The number of satellites and PDOP (c) The ground track (d) The altitude profile

Figure 3: Satellite visibility and flight trajectory



(a) Heading $\psi(t)$ (b) Elevation $\theta(t)$ (c) Bank $\phi(t)$



(d) Heading error (e) Elevation error (f) Bank error

Figure 4: Time series of the three attitude angles [deg] estimated by MC-LAMBDA and provided by the INS

We considered the elevation dependent model (16) with $a_0 = 10$, $\epsilon_0 = 10°$, and $\sigma_0 = 0.3$ m for code conservation and 0.003 m for phase observation. The single-epoch, single-frequency success rate is given in Table 3 showing significant improvement over the unconstrained LAMBDA method. MC-LAMBDA (GPS) estimates for attitude angles are compared with output of an Inertial Navigation System (INS), the Honey-well Laseref II IRS (YG1782B), on board. Figure 4 shows the time series of the three attitude angles from both GPS and IRS. Figures in the second row correspond to difference between the both systems. The standard deviations of these angular differences are provided in Table 3. Except for a few biases, due to aircraft wing deformation during turns, the heading angle can be determined with high precision. Due to the longer baseline along the wing direction, the bank angle is determined more precisely than the elevation angle.

4 CONCLUSIONS

The Multivariate Constrained (MC-)LAMBDA method exploits a priori knowledge of the antenna geometry. This strengthens the observation model and hence improves capacity of fixing the correct set of integer ambiguities. This rigorous inclusion of geometrical constraints enables instantaneous attitude determination using GNSS. In this contribution we demonstrated the effectiveness of the MC-LAMBDA using real data. First we described the GNSS attitude model and the principles of the MC-LAMBDA method. Then, we tested the method using static data from an experiment with low-cost receivers and kinematic data collected during an airborne remote

Single-epoch, single-frequency success rate	LAMBDA [%]	MC-LAMBDA [%]
	28.05	96.42
$\sigma(\psi)$ [deg]		0.06
$\sigma(\theta)$ [deg]		0.15
$\sigma(\phi)$ [deg]		0.10

Table 3: The single-frequency, single-epoch success rate for the LAMBDA and the MC-LAMBDA methods(%) and the standard deviations of the differences between GPS and INS attitude angles (heading ψ, elevation θ and bank ϕ)

sensing campaign. We considered the most challenging namely single-epoch, single frequency unaided ambiguity resolution and attitude determination. The superior success rate performance compared with the unconstrained LAMBDA method even with cheap receivers and high dynamic environment clearly demonstrated the effectiveness of rigorous inclusion of geometric constraints.

Acknowledgments

The second author P.J.G. Teunissen is the recipient of an Australian Research Council Federation Fellowship (project number FF0883188). The flight data set was obtained from Delft University of Technology. An initial version of MC-LAMBDA software was kindly provided by Mr. G. Giorgi from Delft University of Technology. This work is supported by the Australian Space Research Program GARADA project on SAR Formation Flying. All these supports are gratefully acknowledged.

References

Boon, F. and Ambrosius, B. (1997). Results of real-time applications of the LAMBDA method in GPS based aircraft landings. In *Proceedings KIS97*, pages 339–345.

Buist, P. J. (2007). The baseline constrained LAMBDA method for single epoch, single frequency attitude determination applications. In *Proceedings of the 20th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 2007)*, pages 2962 – 2973, Fort Worth, TX, USA.

Cohen, C. (1992). *Attitude Determination Using GPS*. PhD thesis, Stanford University.

Crassidis, J. L. and Markley, F. L. (1997). New algorithm for attitude determination using Global Positioning System signals. *Journal of Guidance, Control, and Dynamics*, 20(5):891–896.

Euler, H.-J. and Goad, C. (1991). On optimal filtering of GPS dual frequency observations without using orbit information. *Journal of Geodesy*, 65:130–143.

Giorgi, G. and Buist, P. (2008). Single-epoch, single-frequency, standalone full attitude determination: experimental results. In *Proceedings of the 4th ESA Workshop on Satellite Navigation User Equipment Technologies, NAVITEC*, ESA-ESTEC, The Netherlands.

Giorgi, G. and Teunissen, P. J. G. (2010). Carrier phase GNSS attitude determination with the multivariate constrained LAMBDA method. In *Aerospace Conference, 2010 IEEE*, pages 1 –12.

Giorgi, G., Teunissen, P. J. G., and Buist, P. J. (2008). A search and shrink approach for the baseline constrained LAMBDA method: Experimental results. In Yasuda, A., editor, *Proceedings of International Symposium on GPS/GNSS*, pages 797 – 806, Tokyo University of Marine Science and Technology.

Giorgi, G., Teunissen, P. J. G., Verhagen, S., and Buist, P. J. (2010). Testing a new multivariate GNSS carrier phase attitude determination method for remote sensing platforms. *Advances in Space Research*, 46(2):118 – 129.

Huang, S., Wang, J., Wang, X., and Chen, J. (2009). The application of the LAMBDA method in the estimation of the GPS slant wet vapour. *Acta Astronomica Sinica*, 50:60–68.

Park, C. and Teunissen, P. J. G. (2009). Integer least squares with quadratic equality constraints and its application to GNSS attitude determination systems. *International Journal of Control, Automation and Systems*, 7(4):566–576.

Psiaki, M. L. (2006). Batch algorithm for global-positioning-system attitude determination and integer ambiguity resolution. *Journal od Guidance, Control, and Dynamics*, 29(5):1070–1079.

Teunissen, P. J. G. (1995). The least-squares ambiguity decorrelation adjustment: a method for fast gps integer ambiguity estimation. *Journal of Geodesy*, 70:65–82.

Teunissen, P. J. G. (1999). An optimality property of the integer least-squares estimator. *Journal of Geodesy*, 73:587–593.

Teunissen, P. J. G. (2007). A general multivariate formulation of the multi-antenna GNSS attitude determination problem:. *Artificial Satellites*, 42(2):97–111.

Verhagen, S. and Teunissen, P. J. G. (2006). New global navigation satellite system ambiguity resolution method compared to existing approaches. *Journal of guidance, control, and dynamics*, 29(4):981–991.