SINGLE-FREQUENCY GPS/SBAS RTK POSITIONING USING AUGMENTATION ADAPTIVE EXTENDED KALMAN FILTER

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KEY WORDS : RTK, SBAS, adaptive EKF

ABSTRACT: Using low-cost single frequency GPS receiver for kinematics positioning, the data takes a long time (comparing to the usual dual-band receivers at present) for solving ambiguity. The paper added SBAS satellite data to improve the problem. SBAS satellite signal strength is weaker and the larger orbit error than GPS satellite. It is necessary to carefully consider the differences between GPS and SBAS satellite data, or it will become unstable or even fail to solve. This paper presents an adaptive EKF theory. The data of master station is used to estimate the error of each satellite. The error information is designed for dynamically adjusting the corresponding system and measurement model of EKF. Validation with the 24-hour experimental data of short baseline, this method can be successfully carried out GPS / SBAS RTK positioning. The success rate of ambiguity resolution of 10 minutes GPS data is improved from 56.9% to 81.9%, when added the SBAS data.

INTRODUCTION：

Real-Time Kinematic (RTK) with single-frequency GNSS receiver, its positioning accuracy is up to centimeter level [1-2]. Without the codes and high-precision dual-frequency phase data, the baseline length is limited (typically <10km) and ambiguity resolution takes longer. The shortcomings of baseline length limited could be improved by VRS technology [3], this study's objective is to increase the efficiency of ambiguity resolution. Ambiguity resolution initial period is related with satellite geometry and measurement accuracy. Generally, dual-frequency measuring instrument with its commercial grade software, the resolution initial period is about five minutes or less [4], as well as the initial period of single-frequency Trimble instrument is about 45 minutes [5]. To reduce the time required to solve the ambiguity of the single-frequency GNSS receiver, many studies adopt INS-assisted single-frequency GNSS receiver for ambiguity resolution work [2,6,7], in order to reduce the number of unknown parameter, to enhance the effectiveness of ambiguity resolution.

SBAS and GPS satellite signals are similar; the same L1 carrier frequency signal with CA code, so the low-cost GNSS receivers can receive the SBAS satellite signals. Most regions in the world received at least 2 SBAS satellite of signals. While GPS satellite geometry is poor, SBAS satellite data can significantly improve the satellite geometry, and thus reduce the time required ambiguity resolution. Compared to the GPS satellites, SBAS satellite signal-to-noise is relatively low and the error from the satellite (orbit, clock error) are larger and unstable. GNSS positioning must appropriately consider the error from the SBAS satellite, otherwise it will create solution instability or divergence. This study proposes estimation error of master station by adding the adaptive Kalman filter correction factor to solve this problem.

2. THEORY BACKGROUND

2.1 GPS measurement

In this paper, single-frequency L1 GPS receivers for data processing, the concept of measuring the phase of L1 and codes, the original observation equation can be written as [8]：

$$\varphi_{1p}^{g} = \frac{f_1}{c}(R_p^g + T_p^g - I_p^g) + f_1(dt_p - dt^g) - N_{1p}^{g} + v_{\varphi_{1p}^g} \qquad (1)$$

$$\rho_{1p}^{g} = R_p^g - c\cdot dt_p + c\cdot dt^g + I_p^g + T_p^g + v_{\rho_p^g} \qquad (2)$$

φ_{1p}^{g} = phase measurement from p receiver to g satellite (units of cycles)； ρ_{1p}^{g} = pseudorange from p receiver to g satellite (units of meters); R_p^g geometric range from p receiver to g satellite (units of meters); T_p^g = tropospheric delay (units of meters); dt_p , dt^g = p receiver and g satellite clock error, respectively (units of meters); I_p^g = ionospheric delay (units of meters); $N_{1p}^{\ g}$ = phase ambiguity (units of cycles); $v_{\varphi_{1p}^{g}}$ = phase noise (units of cycles); $v_{\rho_{1p}^{g}}$ = pheudorange noise (units of meters).

Double difference observation equation can be written as：

$$\varphi_{1pq}^{\ gh} = \frac{f_1}{c}(R_{pq}^{gh} + T_{pq}^{gh} - I_{pq}^{gh}) - N_{1pq}^{\ gh} + v_{\varphi_{1pq}^{gh}} \quad (3)$$

$$\rho_{1pq}^{\ gh} = R_{pq}^{gh} + T_{pq}^{gh} + I_{pq}^{gh} + v_{\rho_{1pq}^{gh}} \quad (4)$$

2.2 Kalman filter

Kalman filter is widely adopted for GPS kinematic positioning. Its observation equation and dynamic equation can be written as：[9]：

$$x_k = \mathbf{T}_{k-1}x_{k-1} + w_{k-1} \ , w_{k-1} \sim N(0, \mathbf{Q}_{k-1}) \quad (5)$$

$$z_k = \mathbf{H}_k x_k + v_k \ , v_k \sim N(0, \mathbf{R}_k) \quad (6)$$

Equation (5) is dynamic equation, and (6) Equation is observation equation, while $\mathbf{x}_k, \mathbf{x}_{k-1}$ represent system state vector at k and k-1epoch，T_{k-1} is Parameter transformation matrix at *k-1* epoch, w_{k-1} is noise of dynamic model. $N(0, \mathrm{Q}_{\mathrm{k-1}})$ on behalf of the Gaussian distribution (mean 0 and standard deviation of $\mathbf{Q}_{k-1}$). $\mathbf{H}_k$ is design matrix, while v_k is the noise vector, $N(0, \mathbf{R}_{k-1})$ on behalf of the Gaussian distribution (mean 0 and standard deviation of $\mathbf{R}_{k-1}$). Kalman filter estimates are divided into the following two steps the time update and the measurement update, the corresponding equation for the time update is：

$$x_k(-) = \mathbf{T}_{k-1}x_{k-1}(+) \quad (7)$$

$$\mathbf{P}_k(-) = \mathbf{T}_{k-1}\mathbf{P}_{k-1}(+)\mathbf{T}_{k-1}^T + \mathbf{Q}_{k-1} \quad (8)$$

$\mathbf{P}_k$ is covariance matrix of system state at k epoch, (-) represents the predicted results and (+) represents the updated results. Measurement update equation is:：

$$x_k(+) = x_k(-) + \mathbf{K}_k[z_k - \mathbf{H}_k x_k(-)] \quad (9)$$

$$\mathbf{P}_k(+) = [\mathbf{I} - \mathbf{K}_k\mathbf{H}_k]\mathbf{P}_k(-) \quad (10)$$

$$\mathbf{K}_k = \mathbf{P}_k(-)\mathbf{H}_k^T[\mathbf{H}_k\mathbf{P}_k(-)\mathbf{H}_k^T + \mathbf{R}_k]^{-1} \quad (11)$$

2.3 Adaptive Kalman filter

Kalman filter solution, i.e., dynamic mode, the error of observation mode and the corresponding statistical parameters must be determined using a priori information. This information is difficult to practice full compliance with all the circumstances. The use of the incomplete information may cause large errors and even divergent solution. Adaptive filter is the role of methodology to reduce the difference between the actual data and the a priori information. The main approach is to estimate the difference between the immediate actual data and the set of statistical parameters, and then correct the statistical parameters with the actual data. Generally we often adopt the concept of innovation sequence. Innovation sequence represents the differences between obtained observations and the actual estimate state, as the following formula [10]:

$$\eta_k = z_k - z_k(-) \quad (12)$$

where $z_k(-) = H_k\, x_k(-)$

Innovation sequence times Kalman gain is called weighted innovation, it is equal to the correction value in equation (9) ($K_k(z_k - z_k(-))$), i.e. the measurement update step, from the estimated state $x_k(-)$ transition to the update status $x_k(+)$of the correction value. The innovation covariance of Kalman filter is as:

$$C_k = E[\eta_k\eta_k^T] = H_kP_k(-)H_k^T + R_k \quad (13)$$

C_k is composed of covariance matrix of prediction state and measurement noise covariance, its value was estimated on the grounds of a priori information about the theoretical value. Using the following formula can be obtained by the actual situation of the innovation covariance [11]:

$$\bar{C}_k = \frac{1}{M-1}\sum_{i=k-M+1}^{k}\eta_i\eta_i^T \quad (14)$$

where M is a window size. Larger M the better average results obtained, the smaller M can filter faster response time.The scalar variable α_k is used to describe the degree of change between $\bar{C}_k$ and C_k：

$$\alpha_k = \max\{1, \frac{1}{m}\mathrm{tr}(\bar{C}_k C_k^{-1})\} \quad \text{or} \tag{15}$$

$$\alpha_k = \max\{1, \frac{\mathrm{tr}(\bar{C}_k)}{\mathrm{tr}(C_k)}\}$$

where m is dimension of z_k.

The unaccounted error is caused by incomplete information of dynamic model or measurement equation. Measurement equation error of the SBAS satellite instability is the bias which this study will overcome.As well as, the effects of incomplete information in the measurement equation can be compensated by the decrease of the magnitude of kalman gain, which is as [11]：

$$\bar{K}_k = \frac{1}{\alpha_k} K_k \tag{16}$$

2.4 Reference-station-assisted adaptive Kalman filter

2.4.1 Estimated error of Reference -station-assisted

The reference station coordinates are known for relative positioning. The known coordinates of the reference station can be used to estimate the amount of error for each satellite, which from the atmospheric effects, satellite error (satellite orbit and satellite clock error) and receiver clock bias dt_i. Equation (2) could be rewritten as：

$$\rho_{1p}^{g} = R_p^g - c \cdot d\,t_p + d_{p,sat}^{g} + d_{p,trop/ion}^{g}(0) \cdot \frac{1}{\cos(z_p^g)} + v_{\rho_{1p}^g} \tag{17}$$

where $d_{p,trop/ion}^{g}(0)$ is the zenith atmospheric error, $\frac{1}{\cos(z_p^g)}$ is mapping function, z_p^g is the zenith angle from receiver p to satellite g. By equation (12), the error of atmosphere and satellite can be estimated. The differential residual error can be estimated by following the formula.

Differential effect of the atmosphere after the following formula can be used for approximate estimates [11]：

$$\Delta E_{trop/ion} = d_{trop/ion}(0) \cdot \left(\frac{1}{\cos(z_q^g)} - \frac{1}{\cos(z_p^g)}\right) \tag{18}$$

Differential satellite error can be approximate the following formula to estimate [12]：

$$\Delta E_{sat} \le \frac{l \cdot d_{sat}}{R} \tag{19}$$

where R is the distance from reference station to the satellite, l is the baseline length from reference station to rover station. ΔE represents overall differential error estimate：

$$\Delta E = \Delta E_{sat} + \Delta E_{trop/ion} \tag{20}$$

2.4.2 Reference-station-assisted adaptive Kalman filter

Reference station data using real-time estimation error value obtained by adding the Kalman filter, called the master-assisted adaptive Kalman filter, the approach is as follows:

σ_i is satellite i observations noise (thermal noise and multipath effects), and $\bar{\sigma}_i$ is the expectations of σ_i (a priori information can be obtained, called the a priori error). Considering $\bar{\sigma}_i$, estimation error of satellite i observations is $\sqrt{\Delta E_i^2 + \bar{\sigma}_i^2}$. We use α_i to present the ratio of a priori error and estimation error:

$$\alpha_i = \frac{\sqrt{\Delta E_i^2 + \bar{\sigma}_i^2}}{\bar{\sigma}_i} \tag{21}$$

when $\alpha_i < k$ (k=3.29, confident level 0.1%), we can determine the systematic error is still within reasonable limits, then we can simply amplify the corresponding R in equation (6). When $\alpha_i \ge 3.29$, the system error is too large, then the exponential function can be used to reduce its value. The application of formulais：

$$e_i = \begin{cases} \sqrt{\Delta E_i^2 + \bar{\sigma}_i^2}, \alpha_i < k \\ k \times e^{\frac{\alpha_i^2}{k^2} - 1}, \alpha_i \ge k \end{cases} \tag{22}$$

By e_i, we can generate an estimated covariance matrix as：

$$C = \begin{bmatrix} {e_1}^2 & & & 0 \\ & {e_2}^2 & & \\ & & \ddots & \\ 0 & & & {e_m}^2 \end{bmatrix} \tag{23}$$

Since double differential mode, the covariance matrix will be as：

$$DDC = B \cdot C \cdot B^T \tag{24}$$

B is coefficient matrix. If the 0-th satellite as the reference satellite, then B is：

$$B = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

Obtained DDC, the equation (11) can be rewritten as：

$$\mathbf{K}_k = \mathbf{P}_k(-)\mathbf{H}_k^T[\mathbf{H}_k\mathbf{P}_k(-)\mathbf{H}_k^T + \mathbf{DDC}_k]^{-1} \tag{26}$$

3. EXPERIMENTAL RESULTS

Experiments were divided into two parts, the first part is the analysis of the codes and phase measurements for the receiver noise. The second part of the SBAS satellite data to assess the added impact of L1 RTK. GPS receiver used in our experiment is developed for navigation, the receiver uses ublox-4t chipset, the receiver costs about $ 100 .The mask angle observed was 15 degrees. In this study, 24 hours a district can receive two SBAS satellites, numbered PRN129 and PRN137 (Japan MSAS satellites).

3.1 Noise analysis of observations

This session using the known coordinates of the station to analyze thermal noise on the code and phase measurements. The data contains 0.5m-length short baseline and a zero-length baseline, observation time was May 7 and May 6, 2011. 24 hours data were analyzed.

For differential model observations ,a pseudo C/N_0 is computed with the following equation [13]：

$$(C/N_0)_{ij} = -10.0 \cdot \log\{\frac{1}{2}\left(10^{-\frac{\frac{C}{N_{0i}}}{10}} + 10^{-\frac{\frac{C}{N_{0j}}}{10}}\right)\} \tag{27}$$

The standard deviation of the zero baseline DD code and phase observations are presented in Fig1 and Fig 2. The horizontal axis of Figure is pseudo C/N_0, each data point for the 120 seconds of data to calculate the standard deviation. GPS satellite signals and SBAS satellite signals will map separately. The figure can be found when C/N_0 = 45 db Hz ,GPS and phase accuracy of the codes were 0.31m and 1.2mm, and SBAS satellites codes and phase accuracy are 1.56m and 6.0mm.

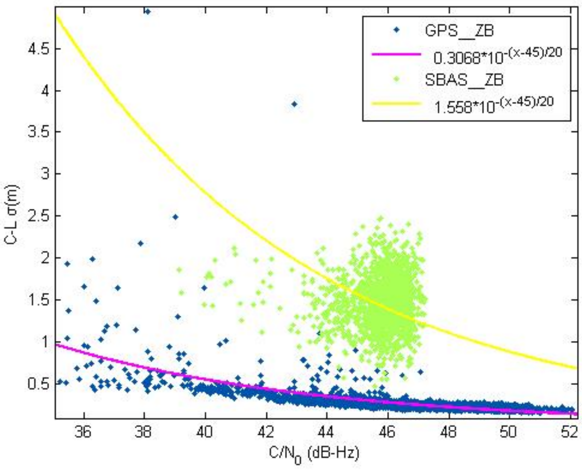

Figure 1 the relation between code error of zero-length baseline and C/N_0

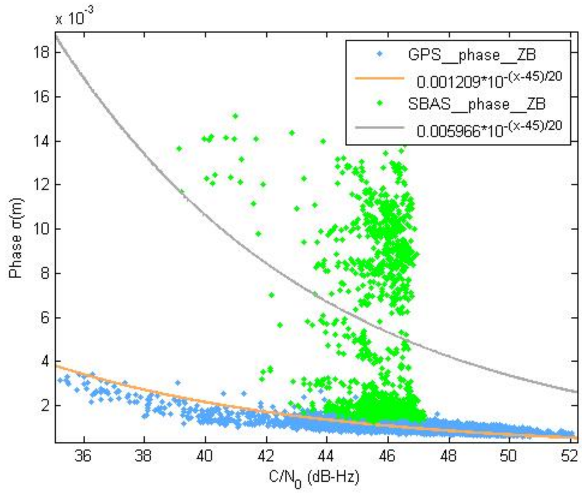

Figure 2 the relation between phase error of zero-length baseline and C/N_0

The standard deviation of the short baseline DD code and phase observations are presented in Fig3 and Fig 4. Can be found from the figure, when C/N_0 = 45 db Hz, GPS codes and phase accuracy were 1.17m and 11mm. And SBAS satellites codes and phase accuracy are 2.86m and 7.3mm, the figure listed in the coefficient can be obtained corresponding to different C/N_0 priori error value, this error value can be used in equation (21) and (22).

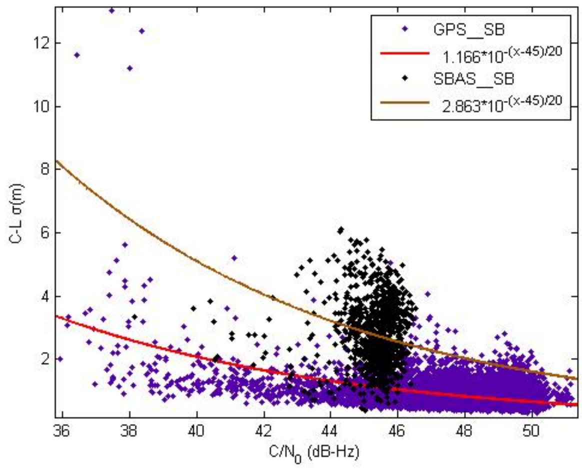

Figure 3 the relation between code error of short-length baseline and C/N_0

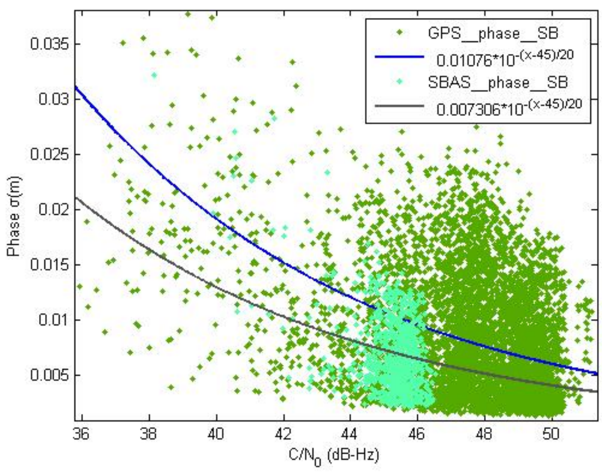

Figure 4 the relation between phase error of short-length baseline and C/N_0

3.2 The success rate of kinematic positioning ambiguity resolution with SBAS satellite

First, true value ambiguity is obtained by using the known baseline vector. Experiments were performed in the Sijhih area of Taiwan, where the length of the baselines was 0.5 m and 3.4 km. The data was collected from the 145st day of 2010. Then one day of test data, every 10 minutes for a period, each period for a solution to work for a total order of 144 solutions. Two minutes, five minutes and ten minutes data is used for ambiguity resolution in each session. Ambiguity resolution success rate is estimated by comparing with the true value.

Table 1 lists the success rate of the three length of time (two minutes, five minutes, ten minutes). From the table of results can be found, adding SBAS satellite data for two minutes, five minutes, ten minutes, improve the success rate of 26.7%, 31.3%, 25.0%. The shorter length of time, the higher the degree of improvement. During the experiment, four sessions every day (length of time period 10-40 minutes) can only be received five GPS satellites. In these periods ambiguity resolution need take a long time. In addition, when any satellite shelter for ambiguity resolution will be difficult work. But with at least two MSAS satellites, it is a great help for the initial ambiguity estimated.

Table 1 the success rate of the three length of time period

Length of time	without SBAS satellite	with SBAS satellite
2 min	31.94% (92/288)	58.68% (169/288)
5 min	43.75% (126/288)	75.00% (216/288)
10 min	56.94% (164/288)	81.94% (236/288)

4. Conclusion and Outlook

Add to SBAS satellite data, can increase the satellite number of two (in Taiwan). For fewer satellite situation, increasing the two satellites is very helpful. However, SBAS satellite signal strength is weak and the larger satellite error. Without considering the two factors may make the solution worse and even cause divergent results. This paper presents a theory can be successfully carried out AAEKF GPS / SBAS positioning, with or without added benefit this method SBAS satellite data in the experiment, improve the success rate of ambiguity resolution level of 25% to 31%. VRS does not currently contain information broadcast SBAS satellite, so join SBAS satellites for positioning the scope is limited to a short baseline (<5km).

REFERENCE

[1] K. Alanen, L. Wirola, J. Käppi, J. Syrjärinne, Mobile RTK: Using Low-Cost GPS and Internet-Enabled Wireless Phones, Inside GNSS Magazine, Vol. 1,No. 4, 2006.(http://www.insidegnss.com)

[2] L. Wirola, K. Alanen, J. Käppi, J. Syrjärinne, Bringing RTK to Cellular Terminals Using a Low-Cost Single-Frequency AGPS Receiver and Inertial Sensors, IEEE/ION PLANS 2006 Conference, San Diego, CA, pp. 24-27, 2006.

[3] Y. Feng, C. Rizos, Network-based geometry-free three carrier ambiguity resolution and phase bias calibration, GPS Solutions, vol.13, pp. 43-56, 2009.

[4] User Menu SKI Software.

[5] Trimble GeoXT handheld datasheet.

[6] H . Wu, On-The-Fly ambiguity resolution with inertial aiding, Master's thesis, University of Calgary, Canada,171 p,2003.

[7] J. Skaloud, Reducing the GPS ambiguity search space by including inertial data. Proc. ION GPS-1998. Nashville, September 15-18, pp. 2073-2080,1998.

[8] Leick, A. *GPS satellite surveying*, 3rd ed., John Wiley & Sons, Inc., Hoboken, 2004; pp.172-177.

[9] A. Gelb, Applied Optimal Estimation. Cambridge, MA: MIT Press, 1974.

[10] R. K. Mehra, Approaches to adaptive filtering, IEEE Transactions on Automatic Control, Vol.AC-17,pp. 693-698, 1972.

[11]K. H. Kim, J. G. Lee, C. G. Park, Adaptive Two-Stage Extended Kalman Filter for a Fault-Tolerant INS-GPS Looselyly Coupled System, IEEE Transaction on Aerospace and Electronic System, Vol. 45, No. 1, pp. 125-137, 2009.

[12] G. Beutler, et al., Atmospheric Refraction and other Important Biases in GPS Carrier Phase Observations , Monograph 12, School of Surveying, University of New South Wales, Kensington, 1988.

[13] B. W. Parkinson and J. J. Spilker Jr., Global Positioning System : Theory and Applications Volume 2, John Wiley & Sons, Inc., 1996, pp 12-23.

[14] P. F.de Bakker, Hans van der Marel, C.J.M. Tiberius, Geometry-free undifferenced, single and double differenced analysis of single frequency GPS,EGNOS and GIOVE-A/B measurement, GPS Solution, vol. 13,pp 305-314,2008.