# RESIDUAL TENSOR ANALYSIS FOR QUALITY ASSESSMENT OF DATA INTEGRATION

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Abstract: Fusion of different data sets in the geospatial field is often used in order to complement the advantages and disadvantages of individual data sets, such as integration of image data and vector data, fusion of Photogrammetric data and LiDAR data, or combination of cadastral data and image data etc. An important step in the data fusion is to register these data sets to a common coordinate system. In this paper, we use residuals after registration to develop a novel quality assessment method, namely residual tensor analysis, for fusion of LiDAR and topographic map data. The results show that our residual tensor method is superior to the common fitting error analysis. In particular, our residual tensor is very useful as a quality indicator for automatic building model construction using LiDAR and topographic map data.

## INTRODUCTION

In geospatial science different data sets from different sources are often integrated in order to find out physical or geometric characteristics which may not be found while using a single date set. The integration of different spatial data sets or temporal data sets is also very useful for the detection of changes in urban, land cover, land-use, buildings, resource distributions, road and communication links, and economic and commerce activities. These data sets may be homogeneous or heterogeneous data, for example photogrammetric images and LiDAR data, land-use data sets at different time epochs, or historic cadastral data sets etc. Since various data sources may have their own coordinate systems including spatial and temporal, the integration of these data sets may need a data registration step to determine the transformation between data sets in order to match corresponding spatial or even temporal relationship. The determination of the transformation for a registration is a critical step in data fusion. Two approaches, namely coordinate-based and sensor-invariant feature-based trans- formations are often used for registration of two or multiply data sets of different kinds. These transformations are performed based on the mathematical principles. The least squares (LS) adjustment is one of commonly used methods. After finishing the transformation, a set or multiply sets of residuals of the corresponding features in these data sets are often produced. In this paper, the residual tensor analysis based on these residuals is developed to assess the quality of the registration. This method is also able to point out the feature or characteristic changes between two data sets. As an example, we integrate LiDAR and topographic map data for automatic building model reconstruction to demonstrate the advantages of the residual tensor analysis.

#### **RESIDUAL TENSOR**

A real symmetric tensor *T* of second order can be described by a matrix as follows:

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$$T = \vec{n} \ \vec{n}^T \tag{1}$$

where  $\vec{n}$  is a vector. Based on the tensor decomposition, the eigenvalues and eigenvectors of a tensor can be obtained. As an example, the tensor constituted by a two-dimensional vector  $\vec{n} = \begin{bmatrix} n_1 & n_2 \end{bmatrix}^T$  can be decomposed in the following form:

$$T = \begin{bmatrix} n_1 n_1 & n_1 n_2 \\ n_2 n_1 & n_2 n_2 \end{bmatrix} = \begin{bmatrix} \vec{e}_1 & \vec{e}_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} \vec{e}_1^T \\ \vec{e}_2^T \end{bmatrix}$$
(2)

with the eigenvalues  $\lambda_1 \ge \lambda_2 = 0$ . The tensor T is positive semi-definite and has a rank of one.

When we register or fit two spatial data sets together using a mathematical method like the LS method or other transformation methods, the residual vector  $\vec{v}$  may be estimated. Normally, we assess the entire registration quality using the transformation results, such as the standard deviation in LS. As mentioned in You and Lin (2011), such a global error indicator like the standard deviation is not suitable for describing the registration quality of individual objects. If we have one object (for example, a roof patch of a building) with residual vectors  $\vec{v}_i$  ( $i = 1, \dots, m$ ) at *m* points or pixels after registration, the residual tensor  $T_i$  at every points can be generated by  $T_i = \vec{n}_i \ \vec{n}_i^T$  and all residual tensors are then added together to form a resultant residual tensor

$$T = \sum_{i=1}^{m} T_i = \sum_{i=1}^{m} \vec{v}_i \vec{v}_i^T$$
(3)

For a closed region, the resultant residual tensor has a rank of two in most cases if we study a two-dimensional case, while it has a rank of three if we study a three-dimensional case. From the visualized geometric point of view, the eigenvectors  $\vec{e}_1$  and  $\vec{e}_2$  describe the orientation of an ellipse with the lengths of axes  $\sqrt{\lambda_1}$  and  $\sqrt{\lambda_2}$  in two-dimensional cases, while the eigenvectors  $\vec{e}_1$ ,  $\vec{e}_2$ , and  $\vec{e}_3$  describe the orientation of an ellipsoid with the lengths of axes  $\sqrt{\lambda_1}$ ,  $\sqrt{\lambda_2}$ , and  $\sqrt{\lambda_3}$  in three-dimensional cases. The orientation described by the eigenvectors are related to the direction of change or discrepancy in the object between data sets, while lengths of axes are related to the size of change.

## AN APPLICATION OF RESIDUAL TENSOR ANALYSIS

In this section, we illustrate five basic residual tensor types after registration of two data sets in two-dimensional cases: (1) Displacement: The spatial region of the new data set is shifted. (2) Reduction: The spatial region of the new data set is enlarged; (4) Increase or decrease: The number of the objects of the new data set is increased or decreased; (5) Mixed cases. If two data sets have any change, we can produce the residual tensors for every closed object. In Figure 1, the black ellipses are derived from the residual tensors. As seen in Figure 1, the major axis of each ellipse approximately points toward the direction of the significant discrepancy between two data sets.



Figure 1: The residual tensor analysis

We apply the residual tensor analysis in the integration of an airborne LiDAR data set (350×500 m<sup>2</sup>) and a data set from the topographic map with scale 1:1000 in order to reconstruct 3D building models. The datums of both data sets are different. The LiDAR data set is referred to the Taiwan geodetic datum 1997.0, and the topographic map is referred to the Taiwan geodetic datum 1967 (You and Hwang, 2006). The boundary points of buildings from LiDAR data sets was extracted by using the TIN method, while the outlines of buildings was obtained from the data set of topographic map and each building outline was saved as a single polygon. All buildings with these two data are put together into the adjustment system. The robust least squares adjustment method developed by the Stuttgart University (Klein and Förstner, 1984) is adopted for the registration of both data sets. After registration, the residual tensor of each building is calculated according to the method mentioned in the previous section. If the residual ellipse is bigger than the tolerance circle, which is given in advance, an obvious discrepancy or change between the boundary points and the outlines of that building exists. In our cases, these discrepancies can be divided two categories: (1) Boundary points from the LiDAR data set are not enough to describe the building outlines. For example, the part of roof faces of a building are covered by trees (Figure 2-a). (2) Some building outlines are lack. For example, the border line between two adjacent buildings is normally not drawn in topographic maps. Another example is that stairwells on the roofs of a building in the topographic map may be not shown (Figure 2-b).

To set up a suitable tolerance circle needs to consider the professional knowledge, science principles, social

economical conditions, law constraints, and natural environmental conditions etc. according to the different integration problems. In our integration of LiDAR and topographic map data sets, we set twice the a posteriori standard deviation of unit weight as the tolerance circle, since it causes a small Type II error. This error type should be avoided in an automatic process of building reconstruction, since we have no chance to find the building without manual examination for all buildings. In our case, the Type II error means that the building is actually changed but identified as unchanged one After residual tensor analysis, all obvious discrepancies or changes between the boundary points and the outlines of the buildings are identified (Figure 3). This is very important since these obvious discrepancies or changes may result in incorrect reconstruction of 3D building models. The residual ellipse of every building therefore is an indicator to assess the quality of registration.



Figure 2: Integration cases and Residuals ellipses



Figure 3: The big residual ellipses in the test area

## Conclusions

In this paper, we have presented the residual tensor analysis to evaluate the quality of registration of two different data sets. Normally, the quality assessment of an integration results may be used the a posteriori standard deviation if using the LS principle. But the a posteriori standard deviation is a global quality indicator of the adjustment. However, it is not appropriate to describe the change of individual polygon, such buildings, and regions etc. Our residual tensor analysis have provided a superior registration quality indicator. Our method developed here is not only useful for the integration of LiDAR and topographic map data sets, but also can be used in the change study of land cover, land-use, urban development, and forest areas etc.

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