MITIGATING THE SYSTEMATIC ERRORS OF e-GPS LEVELING USING NEURAL NETWORK METHOD

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Abstract: e-GPS (Electronic Global Positioning System), which is used in Taiwan, is a type of real-time kinematic satellite positioning technology, such as VRS-RTK (Virtual Reference Station Real Time Kinematic). The height difference between ellipsoidal height h and orthometric height H is called undulation N. For a point p, if the values of h from e-GPS and N from regional geoid model are known, the H value of point p can be calculated using the following equation: $H = h - N$. This is the basic principle of e-GPS leveling. Data analysis of test results revealed that the estimated orthometric height from e-GPS leveling may have systematic errors. This paper proposes a BP (back-propagation) neural network and BP neural network method (BP&BP) method to manage the systematic errors of the estimated orthometric height $\hat{H}$ from e-GPS leveling. The main goal of the proposed method is to mitigate the systematic errors of orthometric height $\hat{H}$ from e-GPS leveling efficiently. Subsequently, the e-GPS leveling accuracy may be improved.

Three data sets (including plane coordinates, ellipsoidal height h from static GPS, orthometric height H from first-order leveling, and ellipsoidal height h from e-GPS) of 145 benchmarks from Tainan City, Taiwan, were used to test the proposed method. The test results show that the proposed method can mitigate the systematic errors of orthometric height $\hat{H}$ from e-GPS leveling efficiently. The proposed methods and the detailed test results are presented in this paper.

INTRODUCTION

e-GPS (Electronic Global Positioning System), used in Taiwan, is a kind of real-time kinematic satellite positioning technology as VRS-RTK (Virtual Reference Station Real Time Kinematic).

Because of basing on different vertical datum, any point on the surface of the Earth, its ellipsoidal height h and orthometric height H are different. The height difference between h and H is called undulation N. Suppose it can be ignored that the vertical deflection on ground is very small, then any point on the ground, the relationship among h, H and N, can be represented it with a simple mathematical equation: $\mathrm{h} = \mathrm{H} + \mathrm{N}$ (Hu, et al., 2004; Kavzoglu and Saka, 2005; Kuhar et al., 2001; Stopar et al., 2006) . Therefore, for a point p, if the values of h from e-GPS and N from regional geoid model are known, then H value of point p can be calculated by the following equation: $\mathrm{H} = \mathrm{h} - \mathrm{N}$. This is the basic principle of e-GPS leveling.

For a certain region, if the regional geoid model has been constructed, the undulation of any point $\hat{\mathrm{N}}$ can be estimated by means of interpolation method. If, the accuracy of the estimated value $\hat{\mathrm{N}}$ meets the required accuracy, the orthometric height H of any point can be calculated quickly by means of e-GPS leveling.

In the past, many experts and scholars have been engaged in research for geometric fitting construction of the regional geoid model theme. They used the following geometric fitting methods: conicoid fitting method (Hu et al., 2004; Lin, 2007), neural network method (Hu, et al., 2004; Kavzoglu and Saka, 2005; Kuhar, et al., 2001; Lin, 2007; Stopar et al., 2006), support vector machine (Zaletnyik, et al., 2008) and so on. In their studies, the author(s) applied different geometric fitting methods to construct a regional geoid model, under different regional conditions, and got good results.

In general, due to the complexity of distribution of the geoid, use of geometric fitting to determine the regional geoid model, the selected model always exists model errors or systematic errors. Therefore, how to mitigate or eliminate the model errors or systematic errors of the regional geoid model has also become one of the research topics. The proposed methods to mitigate or eliminate the systematic errors of the regional geoid model are: the geoid model errors treated as additional parameters using the least squares method (Hu and Sun, 2009), the geoid model errors treated as parameters using least squares collocation method (Hu and Sun, 2009), a quadratic surface fitting an BP neural network method (Hu, et al., 2004; Hu and Sun, 2009).

If, on the other hand, the geoid model of a region is available. And the ellipsoidal height h of each benchmark of this region can be measured by e-GPS. Then, each benchmark has two kinds of orthometric height, an announced orthometric height H from governments, and estimated orthometric height $\hat{H}$ ($\hat{H} = h - \hat{N}$) from e-GPS leveling. The difference between the two values is $\Delta H = H - \hat{H}$. Supposed that there are n benchmarks in this region, then, there are n values of ΔH. Those statistics, such as mean square error, standard deviation, mean, etc. (Ghilani, 2010) from n values of ΔH can be used to evaluate the performance of e-GPS leveling.

Through data analysis of test results, it is found that the ΔH standard deviation of all benchmarks is greater than the expected value in the test area, but also the mean of ΔH is not equal to 0.000m. So, it is suspected that $\hat{H}$ from e-GPS leveling may contain systematic errors. Sources of systematic errors may come from the regional geoid model, various height accuracies between different values of h from e-GPS and static GPS, etc. Therefore, three methods, conicoid fitting method (CFM), BP (back-propagation) neural network and BP neural network method (BP&BP), and BP neural network and conicoid fitting method (BP&CFM), are proposed in this paper, in order to mitigate or eliminate the systematic errors of the e-GPS leveling. This paper is divided into four sections, as an introduction for the first section, section two is the description of proposed methods to improve e-GPS leveling accuracy, for test results and discussion in section three, fourth section for the conclusion of this paper.

PROPOSED METHODS TO IMPROVE E-GPS LEVELING ACCURACY

Related Terms Definitions

For ease of describing the proposed methods and test results, the related terms, statistical values, etc. are defined as follows.

Assume the announced orthometric height of a benchmark is H (treated as a true value), and its estimated orthometric height from e-GPS leveling is $\hat{H}$. The difference between H and $\hat{H}$ is defined as:

$$\Delta H_i = H_i - \hat{H}_i, i = 1,2,\cdots,n \tag{1}$$

where $i = 1,2,\cdots,n$,denotes the serial number of benchmarks; n indicates the total number of benchmarks.

Therefore, for an test region, with n benchmarks, after e-GPS leveling, the maximum, minimum, mean, mean square error, and standard deviation (Ghilani, 2010) of n benchmarks' ΔH can be calculated accordingly. Equation (2), (3), and (4), define the mean, standard deviation and mean square error of ΔH respectively.

$$\text{mean} = \Delta\overline{H} = \frac{\sum_{i=1}^{n} \Delta H_i}{n} \tag{2}$$

$$\sigma = \pm\sqrt{\frac{\sum_{i=1}^{n}\left[\left(\Delta H_i - \Delta\overline{H}\right)\times\left(\Delta H_i - \Delta\overline{H}\right)\right]}{n-1}} \tag{3}$$

$$m = \pm\sqrt{\frac{\sum_{i=1}^{n}\left[\left(\Delta H_i\right)\times\left(\Delta H_i\right)\right]}{n}} \tag{4}$$

Assuming the relationship between ΔH and plane coordinates (x, y) of n benchmarks can be expressed by the following equation:

$$\Delta H_i = f(x_i, y_i) + v_i, i = 1,2,\cdots,n \tag{5}$$

where v_i denotes the residual of benchmark I; $f(x_i, y_i)$ is a function which establishes the relationship between a benchmark's ΔH and its plane coordinates. The geometric fitting methods, such as conicoid fitting method, BP neural network method, etc., can be used to determine function $f(x_i, y_i)$.

The following data $P = \{P_1, P_2, \cdots, P_n\}$ from n benchmarks are used to determine the function $f(x_i, y_i)$.

$$P_i = (x_i, y_i, \Delta H_i), i = 1,2,\cdots,n \tag{6}$$

Assume that there are n benchmarks in a test region. These n benchmarks will be divided into three categories, reference points, check points, and validation points respectively. Data from reference points, with n_1 (about 3/4 of total n benchmarks) points, will be used to determine the coefficients of the polynomial function or to train the neural network and estimate the $\delta\hat{H}$ of every reference point's ΔH. With n_2 ($n_2 = n - n_1$, about 1/4 of total n benchmarks) points, data from check points, will be used to evaluate the fitting accuracy of the determined polynomial function or the trained neural network and estimate the $\delta\hat{H}$ of every check point's ΔH. Finally, data from validation points, with n points, will be used to estimate the $\delta\hat{H}$ of every validation point's ΔH.

If the estimated $\delta\hat{H}$ (denoting the systematic errors of $\hat{H}$ from e-GPS leveling) values of n benchmarks are available, the corrected orthometric height $\tilde{H}$ and corrected orthometric height difference $\Delta\tilde{H}$ after the first time systematic errors correction, can be calculated by equations (7) and (8) respectively.

$$\tilde{H}_i = \hat{H}_i + \delta\hat{H}_i, i = 1,2,\cdots,n \tag{7}$$

$$\Delta\tilde{H}_i = H_i - \tilde{H}_i, i = 1,2,\cdots,n \tag{8}$$

If find that there are still some systematic errors, then further assume that the following equation can express the relationship between $\Delta\tilde{H}$ of n benchmarks and their plane coordinates (x, y).

$$\Delta\tilde{H}_i = g(x_i, y_i) + \tilde{v}_i, i = 1,2,\cdots,n \tag{9}$$

where $\tilde{v}_i$ denotes the residual of benchmark i; $g(x_i, y_i)$ is a function which establishes the relationship between benchmark's $\Delta\tilde{H}$ and its plane coordinates (x, y).

The following data $Q = \{Q_1, Q_2, \cdots, Q_n\}$ from n benchmarks are used to determine the function $g(x_i, y_i)$.

$$Q_i = (x_i, y_i, \Delta\tilde{H}_i) \quad i = 1,2,\cdots,n \tag{10}$$

Again, assume that there are n benchmarks in an test region. These n benchmarks will be divided into three categories, reference points, check points, and validation points respectively. Data from reference points, with n_1 (about 3/4 of total n benchmarks) points, will be used to determine the coefficients of the polynomial function or to train the neural network and estimate the $\delta\hat{\hat{H}}$ of every reference point's $\Delta\tilde{H}$. With n_2 ($n_2 = n - n_1$, about 1/4 of total n benchmarks) points, data from check points, will be used to evaluate the fitting accuracy of the determined polynomial function or trained neural network and estimate the $\delta\hat{\hat{H}}$ of every check point's $\Delta\tilde{H}$. Finally, data from validation points, with n points, will be used to estimate the $\delta\hat{\hat{H}}$ of every validation point's $\Delta\tilde{H}$.

If the estimated $\delta\hat{\hat{H}}$ (denoting the systematic errors of $\tilde{H}$) values of n benchmarks are available, the corrected orthometric height $\tilde{\tilde{H}}$ and corrected orthometric height difference $\Delta\tilde{\tilde{H}}$ after the second time systematic errors correction, can be calculated by equations (11) and (12) respectively.

$$\tilde{\tilde{H}}_i = \tilde{H}_i + \delta\hat{\hat{H}}_i, i = 1,2,\cdots,n \tag{11}$$

$$\Delta\tilde{\tilde{H}}_i = H_i - \tilde{\tilde{H}}_i, i = 1,2,\cdots,n \tag{12}$$

The varied statistical values, such as mean square error, standard deviation, etc., of $\Delta\tilde{H}$, $\Delta\tilde{\tilde{H}}$, $\delta\hat{H}$, $\delta\hat{\hat{H}}$ can be computed in the light of calculation of varied statistical values of ΔH. In addition, for simplicity, σ_{ref}, σ_{chk}, σ_{val} represent the standard deviations of reference points, check points, and validation points respectively.

Conicoid Fitting Method (CFM)

The conicoid fitting method (CFM, also known as polynomial fitting) is usually used to construct a regional geoid model (Hu et al., 2004; Hu and Sun, 2009; Lin, 2007). However, CFM will be used to estimate $\delta\hat{H}$. The following polynomial represents the function $f(x_i, y_i)$ of equation (5):

$$f(x_i, y_i) = a_1 + a_2 x_i + a_3 y_i + a_4 x_i y_i + a_5 x_i^2 + a_6 y_i^2 + \cdots \tag{13}$$

where $a_1, a_2, a_3, \cdots$ denotes the undetermined coefficients of a polynomial. Three types of CFM will be tested in this paper, i.e. 4-parameter CFM (a polynomial with undetermined coefficients $a_1, \cdots, a_4$), 6-parameter CFM (a polynomial with undetermined coefficients $a_1, \cdots, a_6$), and 10-parameter CFM (a polynomial with undetermined coefficients $a_1, \cdots, a_{10}$). When the total number of benchmarks is greater than the number of undetermined coefficients, the undetermined coefficients of a polynomial can be estimated using the least squares method. And, then enter the plane coordinates (x, y) of benchmarks within the region to equation (13), those values, such as $\delta\hat{H}$, $\tilde{H}$, and $\Delta\tilde{H}$,after the first time systematic error correction of e-GPS leveling, can be estimated using the following CFM procedures.

BP Neural Network and BP Neural Network Method (BP&BP)

Back-propagation (BP) neural network (i.e., the multilayer feed-forward neural network), is one of the neural network algorithms. The structure of BP neural network is divided into input layer, hidden layer and an output layer.

BP neural networks are often used to construct a regional geoid model (Hu et al., 2004; Hu and Sun, 2009; Kavzoglu and Saka, 2005; Kuhar et al., 2001; Lin, 2007; Lin, 2012; Stopar et al., 2006). However, this paper will use the BP neural network and BP neural network method (BP&BP) to estimate the values of $\delta\hat{H}$ and $\delta\hat{\hat{H}}$ of e-GPS leveling respectively.

First of all, a $2 \times p_1 \times 1$ BP neural network (2 represents the input layer has two elements, plane coordinates (x, y) of each point; p_1 denotes the number of neurons in the hidden layer; 1 represents the output layer has 1 element, ΔH value of each point), is trained to determine the function $f(x_i, y_i)$ of equation (5), using n benchmarks data $P = \{P_1, P_2, \cdots, P_n\}$. And then enter the plane coordinates (x, y) of points within the region, to calculate $\delta\hat{H}$, $\tilde{H}$, and $\Delta\tilde{H}$ values of all benchmarks, after the first time systematic errors correction of e-GPS leveling.

Next, a $2 \times p_2 \times 1$ BP neural network (2 represents the input layer has two elements, plane coordinates (x, y) of each point; p_2 denotes the number of neurons in the hidden layer; 1 represents the output layer has 1 element, $\Delta\tilde{H}$ value of each point), is trained to determine the function $g(x_i, y_i)$ of equation (9), using n benchmarks data $Q = \{Q_1, Q_2, \cdots, Q_n\}$. And then enter the plane coordinates (x, y) of points within the region, to calculate $\delta\hat{\hat{H}}$, $\tilde{\tilde{H}}$, and $\Delta\tilde{\tilde{H}}$ values of all benchmarks, after the second time systematic errors correction of e-GPS leveling.

BP Neural Network and Conicoid Fitting Method (BP&CFM)

If n benchmarks data $P = \{P_1, P_2, \cdots, P_n\}$ are available, first find the mean $\Delta\overline{H}$ of all points' ΔH, using equation (2). And, then calculate the dH value of each point using the following equation.

$$dH_i = \Delta H_i - \Delta\overline{H}, i = 1, 2, \cdots, n \tag{14}$$

If the following equation can express the relationship between the dH values of n benchmarks and their plane coordinates (x, y).

$$dH_i = h(x_i, y_i) + \tilde{\tilde{v}}_i, i = 1, 2, \cdots, n \tag{15}$$

where $\tilde{\tilde{v}}_i$ indicates the residual of benchmark i.

There are two steps to be followed using BP&CFM. First of all, train a $2 \times p_1 \times 1$ BP neural network (2 represents the input layer has two elements, plane coordinates (x, y) of each point; p_1 denotes the number of neurons in the hidden layer; 1 represents the output layer has 1 element, dH value of each point), using n benchmarks data $(x_i, y_i; dH_i), i = 1,2,\cdots,n$, to determine the function $h(x_i, y_i)$ of equation (15). And then enter the plane coordinates (x, y) of points within the region, to calculate the estimation $\delta d\hat{H}$ of all points' dH. Finally, calculate $\delta\hat{H}$ (using equation (16)), $\widetilde{H}$, and $\Delta\widetilde{H}$ of all benchmarks.

$$\delta\hat{H}_i = \Delta\overline{H} + \delta d\hat{H}_i, i = 1,2,\cdots,n \tag{16}$$

Next, determine the CFM's 6 polynomial coefficients of function $g(x_i, y_i)$ of equation (9), using the least squares method, with all data $Q = \{Q_1, Q_2, \cdots, Q_n\}$. And then enter the plane coordinates (x, y) of points within the region, to calculate $\delta\hat{\hat{H}}$, $\widetilde{\widetilde{H}}$, and $\Delta\widetilde{\widetilde{H}}$ values of all benchmarks.

TEST RESULTS AND DISCUSSION

Tainan e-GPS System

Tainan City Government has established its e-GPS system in September 2007. The e-GPS system contains 6 reference stations, and covers the whole city. Five reference stations, SCES, NJES, RFES, WHES, BKBL, evenly distributed in Tainan city's borders, forming a nearly regular pentagon network; and the approximate geographic center in Tainan City setting of the sixth reference station, KAWN, its location just in the pentagonal-shaped center. And, it makes all the distances between the reference stations less than 30 km. In order to improve the accuracy and efficiency of e-GPS surveying in the mountain area, the seventh reference station, YJLO, was installed in April 2010. Hence, the Tainan e-GPS system has 7 reference stations since then. All reference stations are equipped with Trimble NetR5, and the mobile stations are equipped with Trimble R8. Both types of receivers, Trimble NetR5 and Trimble R8, can track signals from GPS satellites and GLONASS satellites. The distribution map of 7 reference stations of Tainan e-GPS system is shown in Figure 1.

Tainan e-GPS system, through the field testing, achieving the following accuracies: ±2cm in plane coordinates (x, y), and ±5cm in ellipsoidal height h, its accuracy is sufficient to be applied to the cadastral surveying, engineering surveying, etc. (Tainan, 2012).

Test Data

Three data sets of Tainan area (with total area of about 2,192 square kilometers or 219,200 hectares) are used to test the proposed methods. The data sets including: (1) data set 1 of 145 first-order benchmarks, with orthometric height H from first-order leveling and plane coordinates (x, y) and ellipsoidal height h from static GPS surveying of 2003, provided by the Ministry of the Interior, Republic of China; (2) data set 2 of 145 first-order benchmarks, with orthometric height H only from first-order leveling of 2009, provided by the Ministry of the Interior, Republic of China; (3) data set 3 of 118 first-order benchmarks, with plane coordinates (x, y) and ellipsoidal height h from Tainan e-GPS system of 2011, provided by Tainan City Government.

Test Results and Discussion

Accuracy Analysis of e-GPS Leveling: The following procedures are performed to evaluate the accuracy of e-GPS leveling: (1) Train a $2 \times p_1 \times 1$ BP neural network (2 represents the input layer has two elements, plane coordinates (x, y) of each benchmark; p_1 denotes the number of neurons in the hidden layer; 1 represents the output layer has 1 element, undulation N of each benchmark), in order to construct a regional geoid model of Tainan City, with 145 first-order benchmarks of data set 1; (2) Estimate undulation $\hat{N}$ of all 118 first-order benchmarks of data set 3, using the trained $2 \times p_1 \times 1$ BP neural network and the plane coordinates (x, y) of each benchmark; (3) Calculate the orthometric height $\hat{H}$, using the formula of $\hat{H} = h - \hat{N}$, with the ellipsoidal height h from e-GPS system and the estimated undulation $\hat{N}$ from the above procedure, of all 118 first-order benchmarks of data set 3; (4) Compute the height difference ΔH, using the formula of $\Delta H = H - \hat{H}$ (H denotes the orthometric height from data set 2, and $\hat{H}$ represents the estimated orthometric height from procedure 3), of all 118 first-order benchmarks of data set 3.

According to the preceding procedure 1, in order to construct a regional geoid model of Tainan City with BP neural network, 145 first-order benchmarks of data set 1 are divided into two groups, one group as the reference point (109 points) to train a BP neural network; another group as a check point (36 points) to assess the accuracy of the regional geoid model.

Since orthometric height H and ellipsoidal height h of each benchmark of data set 1 are known, the undulation N of each benchmark can be calculated using the formula $N = h - H$. And, it is assuming that N is the true value. Suppose further that the undulation of each benchmark estimated by the trained BP neural network is $\hat{N}$, then, the undulation difference ΔN of each benchmark, is defined by the following equation.

$$\Delta N_i = N_i - \hat{N}_i, i = 1,2,\cdots,n \tag{17}$$

where $i = 1,2,\cdots,n$ stands for the sequential number of check points; n is the total number of check points.

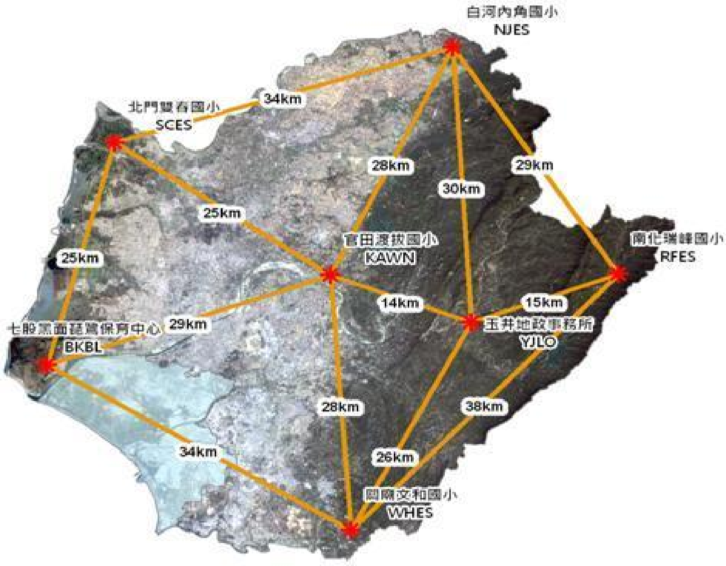

Figure 1. The distribution map of 7 reference stations of Tainan e-GPS system.

After trial and error tests, it is found that a $2 \times 35 \times 1$ BP neural network can offer better regional geoid model accuracy (Lin, 2007; Lin, 2012). The statistics of ΔN of 36 check points of data set 1 are shown in Table 1. In Table 1, 'm (m)' indicates mean square error in units of meter; 'σ (m)' indicates standard deviation in units of meter; 'Mean (m)' indicates mean value in units of meter; 'Maximum (m)' indicates maximum value in units of meter; 'Minimum (m)' indicates minimum value in units of meter.

Table 1. The statistics of ΔN of 36 check points of data set 1, using a geoid model from $2 \times 35 \times 1$ BP neural network.

m (m)	σ (m)	Mean (m)	Maximum (m)	Minimum (m)
±0.029	±0.028	0.009	0.089	-0.054

Based on the previously mentioned procedures 2 to 4, compute the height difference ΔH of all 118 first-order benchmarks of data set 3. The statistics of ΔH of all 118 first-order benchmarks are shown in Table 2. It can be seen from the results of Table 2 that the standard deviation of ΔH is ± 0.050m.

Table 2. The statistics of ΔH of 118 first-order benchmarks of Tainan City.

m (m)	σ (m)	Mean (m)	Maximum (m)	Minimum (m)
±0.072	±0.050	-0.051	0.061	-0.213

The accuracy of h from e-GPS system is ± 0.050m (Tainan, 2012). Besides, the accuracy of estimated undulation $\hat{N}$ is ± 0.028m, according to the results of Table 1. Based on the formula $\hat{H} = h - \hat{N}$ and according to the principle of error propagation, the accuracy of $\hat{H}$ from e-GPS leveling is ±0.057m.

By definition of $\Delta H = H - \hat{H}$, where the accuracy of H is ±0.009m (Yang et al., 2003); the accuracy of $\hat{H}$ is ±0.057m. According to the principle of error propagation, the accuracy of ΔH from e-GPS leveling is ±0.058m.

Therefore, further examining the results of Table 2, it is found that (1) the standard deviation and mean square error of ΔH varies considerably (0.022m), and (2) the mean value of ΔH is -0.051m (not 0.000m). Therefore, judging the test results of the e-GPS leveling, it may still have some systematic errors to be corrected.

Test Results of Proposed Methods: $P=\{P_1,P_2,\cdots,P_n\}$ and $Q=\{Q_1,Q_2,\cdots,Q_n\}$ data of 118 first-order benchmarks of data set 3, will be used to test the three proposed methods. The number of reference points n_1, check points n_2 and validation point n of data set 3 are 89, 29, and 118 respectively.

A. Test Results of CFM

Based on the above-mentioned procedures of CFM, data of 118 first-order benchmarks are used to test the performances of 4-parameter, 6-parameter, and 10-parameter CFM. The statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM, are shown in Table 3. In Table 3, $\Delta H(N/A)$ denotes the value of ΔH before correcting systematic errors; $\Delta\widetilde{H}(4-par)$, $\Delta\widetilde{H}(6-par)$, and $\Delta\widetilde{H}(10-par)$ denote the value of ΔH after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM respectively.

Can be seen from the results in Table 3, after correcting systematic errors estimated by 4-parameter CFM, the standard deviation of $\Delta\widetilde{H}$ decreased ±0.037m (close to the mean square error value), and the mean of $\Delta\widetilde{H}$ dropped to 0.000m; after correcting systematic errors estimated by 6-parameter CFM, the standard deviation of $\Delta\widetilde{H}$ decreased ±0.034m (close to mean square error), and the mean of $\Delta\widetilde{H}$ dropped to 0.000m; after correcting systematic errors estimated by 10-parameter CFM, the standard deviation of $\Delta\widetilde{H}$ decreased ±0.028m (With mean square error differ by ± 0.002m), and the mean of $\Delta\widetilde{H}$ dropped to- 0.011m.

Table 3. The statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM

ΔH	m (m)	σ (m)	Mean (m)	Maximum (m)	Minimum (m)
$\Delta H(N/A)$	±0.072	±0.050	-0.051	0.061	-0.213
$\Delta\widetilde{H}(4-par)$	±0.036	±0.037	0.000	0.091	-0.136
$\Delta\widetilde{H}(6-par)$	±0.034	±0.034	0.000	0.070	-0.122
$\Delta\widetilde{H}(10-par)$	±0.030	±0.028	-0.011	0.098	-0.087

B. Test Results of BP&BP

Based on the specific procedures of BP&BP, systematic errors of e-GPS leveling, $\delta\hat{H}$ and $\delta\hat{\hat{H}}$, should be estimated by $2\times p_1\times 1$ and $2\times p_2\times 1$ BP neural networks respectively. In order to determine the number of neurons p_1 and p_2 using trial and error method, $P=\{P_1,P_2,\cdots,P_n\}$ and $Q=\{Q_1,Q_2,\cdots,Q_n\}$ data of 118 first-order benchmarks are used to train $2\times p_1\times 1$ and $2\times p_2\times 1$ BP neural networks respectively. In order to demonstrate the procedures of determining the number of neurons p_1 with trial and error method, the statistics of σ_{ref}, σ_{chk} and σ_{val} of $\delta\hat{H}$ of 118 first-order benchmarks, after changing the number of neurons ($p_1=1,2,\cdots,15$) in the hidden layer of $2\times p_1\times 1$ BP neural network, are shown in Table 4. It can be seen from Table 4 that when the number of neurons p_1 is 8 the results are best. Hence, a $2\times 8\times 1$ BP neural network will be used to estimate values of $\delta\hat{H}$, $\widetilde{H}$, $\Delta\widetilde{H}$ of 118 first-order benchmarks.

Then, the number of neurons p_2 of $2\times p_2\times 1$ BP neural network is determined, using trial and error method, with $P=\{P_1,P_2,\cdots,P_n\}$ and $Q=\{Q_1,Q_2,\cdots,Q_n\}$ data of 118 first-order benchmarks. It is found that the number of neurons p_2 is 5 the results are best. Hence, a $2\times 5\times 1$ BP neural network will be used to estimate values of $\delta\hat{\hat{H}}$、$\widetilde{\widetilde{H}}$ and $\Delta\widetilde{\widetilde{H}}$ of 118 first-order benchmarks. The statistics of σ_{ref}, σ_{chk} and σ_{val} of $\delta\hat{\hat{H}}$ of 118 first-order benchmarks, using a $2\times 5\times 1$ BP neural network are shown in Table 5.

Table 6 shows that the statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&BP. In Table 6, $\Delta H(N/A)$ denotes the value of ΔH before correcting systematic errors; $\Delta\widetilde{H}(BP1-2\times 8\times 1)$ and $\Delta\widetilde{\widetilde{H}}(BP2-2\times 5\times 1)$ denote values of ΔH after correcting systematic errors estimated by

$2\times8\times1$ BP neural network and $2\times5\times1$ BP neural network respectively. Be seen from the results in Table 6, after correcting systematic errors estimated by a $2\times8\times1$ BP neural network, the standard deviation of $\Delta\tilde{H}$ decreases to ±0.030m (difference between the standard deviation and the mean square error is ±0.014m), the mean of $\Delta\tilde{H}$ declines to 0.032m. After correcting systematic errors estimated by a $2\times5\times1$ BP neural network, the standard deviation of $\Delta\tilde{\tilde{H}}$ decreases to ±0.029m (equal to the mean square error), and the mean of $\Delta\tilde{\tilde{H}}$ declines to -0.007 m.

Table 4. The statistics of σ_{ref}, σ_{chk} and σ_{val} of $\delta\hat{H}$ of 118 first-order benchmarks, after changing the number of neurons ($p_1 = 1,2,\cdots,15$) in the hidden layer of $2\times p_1\times1$ BP neural network

p_1	1	2	3	4	5	6	7	8
σ_{ref} (m)	0.032	0.028	0.028	0.027	0.028	0.028	0.033	**0.027**
σ_{chk} (m)	0.039	0.034	0.033	0.034	0.034	0.036	0.041	**0.032**
σ_{val} (m)	0.035	0.032	0.030	0.030	0.031	0.032	0.038	**0.029**
p_1	9	10	11	12	13	14	15	
σ_{ref} (m)	0.029	0.028	0.029	0.029	0.028	0.030	0.029	
σ_{chk} (m)	0.036	0.034	0.033	0.033	0.033	0.033	0.034	
σ_{val} (m)	0.033	0.030	0.030	0.031	0.031	0.030	0.031	

Table 5. The statistics of σ_{ref}, σ_{chk} and σ_{val} of $\delta\hat{\hat{H}}$ of 118 first-order benchmarks, using a $2\times5\times1$ BP neural network

σ_{ref} (m)	σ_{chk} (m)	σ_{val} (m)
0.029	0.032	0.029

Table 6. The statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&BP

ΔH	m (m)	σ (m)	Mean (m)	Maximum (m)	Minimum (m)
$\Delta H(N/A)$	±0.072	±0.050	-0.051	0.061	-0.213
$\Delta\tilde{H}(BP1-2\times8\times1)$	±0.044	±0.030	0.032	0.119	-0.063
$\Delta\tilde{\tilde{H}}(BP2-2\times5\times1)$	±0.029	±0.029	-0.007	0.077	-0.105

C. Test Results of BP&CFM

Based on the specific procedures of BP& CFM, systematic errors of e-GPS leveling, $\delta\hat{H}$ and $\delta\hat{\hat{H}}$, should be estimated by a $2\times p_1\times1$ BP neural network and a 6-parameter CFM respectively. In order to determine the number of neurons p_1 using trial and error method, $P=\{P_1,P_2,\cdots,P_n\}$ data of 118 first-order benchmarks are used to train a $2\times p_1\times1$ BP neural network. It is found that the number of neurons p_1 is 2 the results are best. Hence, a $2\times2\times1$ BP neural network will be used to estimate values of $\delta\hat{H}$, $\tilde{H}$, $\Delta\tilde{H}$ of 118 first-order benchmarks. Then, 6 parameters of CFM are estimated, using least squares method, with $Q=\{Q_1,Q_2,\cdots,Q_n\}$ data of 118 first-order benchmarks. Finally, values of $\delta\hat{\hat{H}}$ 、 $\tilde{\tilde{H}}$ and $\Delta\tilde{\tilde{H}}$ of 118 first-order benchmarks are estimated by a 6-parameter CFM.

Table 7 shows statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&CFM. In Table 7, $\Delta H(N/A)$ denotes the value of ΔH before correcting systematic errors; $\Delta\tilde{H}(BP1-2\times2\times1)$ and $\Delta\tilde{\tilde{H}}(CFM2-6-par)$ denote values of ΔH after correcting systematic errors estimated by $2\times2\times1$ BP neural network and 6-parameter CFM respectively.

It can be seen from the results in Table 7, after correcting systematic errors estimated by a $2\times2\times1$ BP neural network, the standard deviation of $\Delta\tilde{H}$ decreases to ±0.031m (difference between the standard deviation and the mean square error is ±0.011m), the mean of $\Delta\tilde{H}$ declines to 0.029m. Then, after correcting systematic errors estimated by a 6-parameter CFM, the standard deviation of $\Delta\tilde{\tilde{H}}$ decreases to ±0.029m (equal to the mean square error), and the mean of $\Delta\tilde{\tilde{H}}$ declines to 0.000 m.

Table 7. The statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&CFM algorithm.

ΔH	m (m)	σ (m)	Mean (m)	Maximum (m)	Minimum (m)
$\Delta H(N/A)$	±0.072	±0.050	-0.051	0.061	-0.213
$\Delta\tilde{H}(BP1-2\times2\times1)$	±0.042	±0.031	0.029	0.119	-0.079
$\Delta\tilde{\tilde{H}}(CFM2-6-par)$	±0.029	±0.029	0.000	0.066	-0.105

D. Summary

The statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by CFM, BP&BP, and BP&CFM respectively, are summarized and shown in Table 8. In Table 8, N/A stands for the value of ΔH without any systematic error correction; 4-CFM, 6-CFM and 10-CFM denote values of $\Delta\tilde{H}$ after correcting systematic errors estimated by 4-parameter, 6-parameter, and 10-parameter CFM respectively; BP&BP denotes the value of $\Delta\tilde{\tilde{H}}$ after correcting systematic errors estimated by a $2\times8\times1$ BP neural network and a $2\times5\times1$ BP neural network respectively; BP&CFM indicates the value of $\Delta\tilde{\tilde{H}}$ after correcting systematic errors estimated by a $2\times2\times1$ BP neural network and 6-parameter CFM respectively.

Can be seen from the results of Table 8, after systematic error correction estimated by CFM, BP&BP, and BP&CFM, the standard deviation of ΔH can be reduced considerably. Among them, the performances of BP&CFM and BP&BP are the best. In terms of reduced the mean of ΔH, BP&CFM, 4-CFM and 6-CFM perform the best. Then, it is checked that whether the mean square error of ΔH is equal to the standard deviation of ΔH or not? It is found that BP&CFM, BP&BP and 6-CFM meet the requirements. Therefore, on three aspects into consideration, i.e. (1) Is the standard deviation of ΔH the smallest? (2) Is the standard deviation of ΔH equal to the mean square error of ΔH? (3) Is the mean of ΔH is equal to 0.000m? It is found that the performance of BP & CFM is the best, and followed by BP&BP. The ΔH comparison charts of 118 first-order benchmarks, before and after correcting systematic errors estimated by BP&CFM, are shown in Figure 2. In Figure 2, "No Corr." denotes the value of ΔH before correcting systematic errors; "After BP1 Corr." and "After CFM2 Corr." denote values of ΔH after correcting systematic errors estimated by $2\times2\times1$ BP neural network and 6-parameter CFM respectively; the vertical axis expresses the value of ΔH (m); and the horizontal axis stands for the sequential number of 118 first-order benchmarks.

Table 8. The statistics of ΔH of 118 first-order benchmarks, before and after correcting systematic errors estimated by CFM, BP&BP, and BP&CFM respectively.

ΔH	m (m)	σ (m)	Mean (m)	Maximum (m)	Minimum (m)
N/A	±0.072	±0.050	-0.051	0.061	-0.213
4-CFM	±0.036	±0.037	0.000	0.091	-0.136
6-CFM	±0.034	±0.034	0.000	0.070	-0.122
10-CFM	±0.030	±0.029	-0.011	0.098	-0.087
BP&BP	±0.029	±0.029	-0.007	0.077	-0.105
BP&CFM	**±0.029**	**±0.029**	**0.000**	**0.066**	**-0.105**

COMCLUSIONS

Address the systematic errors of estimated orthometric height $\hat{H}$ of e-GPS leveling, three methods, i.e. CFM, BP&BP, and BP&CFM, are proposed in this paper. Three data sets of Tainan City are used to test the proposed methods. The test results show that, among the three methods, BP & CFM is the most effective way to mitigate the systematic errors of e-GPS leveling, followed BP&BP, and 6-parameter CFM. Using BP & CFM algorithm, for example, the standard deviation of ΔH is reduced to ±0.029m from ±0.050m and the mean of ΔH is equal to 0.000m.

In this paper, it is found that the systematic errors of e-GPS leveling can be mitigated effectively if BP&CFM is applied, using the data sets from Tainan City. However, if the test area is increased, such as the southern region of Taiwan, and even extended to the entire island of Taiwan, the BP & CFM algorithm, is still valid? Remains to be further validated in the future

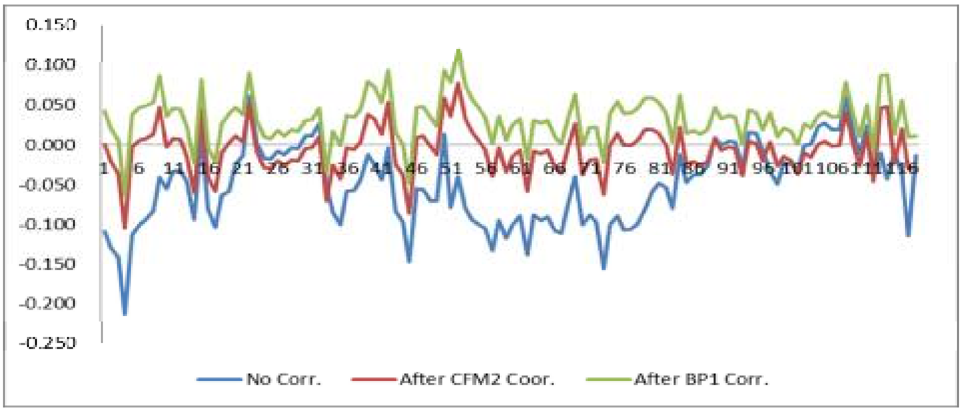

Figure 2. The ΔH comparison charts of 118 first-order benchmarks, before and after correcting systematic error estimated by BP&CFM.

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