ASSESSMENT OF GRAVITY REQUIREMENTS FOR PRECISE GEOID DETERMINATION IN THAILAND

Puttipol DUMRONGCHAI

Assistant Professor, Dept. of Civil Engineering, Chiang Mai University 50200, Thailand;
Tel: +6653-944156-9; Fax: +6653-892376; E-mail: puttipol.d@cmu.ac.th

ABSTRACT: In mountainous areas, the insufficient of gravimetric data limits the determination of an accurate geoid model for height-system modernization using GPS technology that directly relates ellipsoidal heights in a national geodetic datum to orthometric heights in a national vertical datum. This study presents an assessment of gravity data resolution that requires achieving a certain level of omission error in geoid computation for Thailand. Topography-implied gravity anomalies were simulated by residual terrain model (RTM) approach, using SRTM (Shuttle Radar Topography Mission) digital elevation model, and augmented with an existing network of terrestrial gravity data. Analysis of the simulated data through Stokes' integral shows that three-arcminute (~5.5km) spatial resolution can cause 10-cm omission error in term of geoid undulations in rugged terrains. The errors are reduced to a few centimeters if the spatial resolutions are as small as one arcminute (~1.8km) or topographic terrains are relatively flat.

KEYWORDS: Geoid determination, Residual terrain model (RTM), Height modernization, Omission error

INTRODUCTION

In recent years, there has become awareness of the need for an accurate local geoid model in Thailand due to the availabilities of national vertical control networks, new gravimetric quantities, and high resolution digital elevation models. Successful development of the global positioning system (GPS) provides the effective means to acquire very high accurate positions in a geocentric reference frame. Furthermore, the National Geospatial-intelligence Agency (NGA) officially released the latest earth gravity model of 2008 (EGM2008) (Pavlis et al., 2012). The attempt for local geoid modeling has been planned to support the use of GPS surveys to convert GPS-based geodetic heights in the WGS84 national horizontal frame to orthometric heights in Kolak 1915 national vertical datum. Such a height-system modernization plays a role in engineering surveys and mapping applications. However, local geoid errors can reach up to sub-meter levels, mostly in the mountainous areas, devoid of existing gravity data. The conversion of GPS ellipsoidal heights to orthometric heights thus depends on the accuracy and intensity of gravimetric data for geoid determination.

Besides errors related to gravimetric data for geoid modeling, omission error resulting from the lack of resolution in the gravimetric data contributes to the total error in geoid undulation. For instance, the EGM2008 global model, having spherical harmonic coefficients up to 2190 degrees and 2160 orders (corresponding to the spatial resolution of 5 arc-minutes or about 9 km), produces the signal omission error of about 4 cm (Jekeli et al. 2009). The error can be large due to EGM2008 unable to represent high-frequency gravity signals in rugged terrains. To handle the problem of devoid areas of gravity data, Hirt et al. (2010) utilized the method of RTM for computing estimates of the omission error in mountainous areas with insufficient distribution or scarce availability of gravity data. The RTM method, introduced by Forsberg and Tscherning (1981) and Forsberg (1984), considered only local high-frequent topographic irregularities by referring all elevations to a smooth mean elevation surface, e.g., the DTM2006.0 global elevation model (Pavlis et al. 2012). The high-frequency gravity signals were constructed using a digital elevation model, e.g., SRTM. Testing areas were in German Alps. By applying RTM omission error estimates to EGM2008 height anomalies (which can be converted to geoid undulations according to Heiskanen and Moritz (1969, p. 253)), the comparison with GPS/leveling data showed the significant improvement rate of almost 50% better than the case of EGM2008 height anomalies alone. In addition, the method was easily applied without the need of any gravity measurements.

The objectives of this paper are to assess terrestrial gravity requirements for acceptable levels of geoid accuracy with respect to spatial resolutions over the areas, where existing data are not intensified or available. Our main focus of this paper considers only the omission error and neglects other errors associated to data measurements. We adapted the RTM method presented by Hirt et al. (ibid.) for estimating omission errors to the geoid undulations. For the topography-implied gravity anomalies, we used a three-arcsecond digital elevation model (DEM) [e.g. the Shuttle Radar Topography Mission (SRTM) (Javis et al. 2004; Rodriguez et al. 2005): version 4 (void-filled areas) available at http://srtm.csi.cgiar.org/]. The entire computation processes as well as numerical results are discussed. Finally, the conclusions are summarized.

THE TOPOGRAPHY-IMPLIED GRAVITY ANOMALIES USING RESIDUAL TERRAIN MODEL

According to Forsberg (1984), the residual terrain model considers only the surface of topography corresponding to short wavelengths of the earth's gravity field, as shown in Figure 1. A mean elevation surface is introduced as a smooth surface for removing masses above this surface and filling up spaces below. The mean elevation surface can be any digital elevation model, e.g., DTM2006.0 (Pavlis et al. 2012). Under a planar approximation, the RTM gravities, δg_{RTM}, evaluated at point P on the (topographic) surface is given by the integral of form in the triad coordinate system (x,y,z)

$$\delta g_{RTM}(P) = k\rho \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{z_1=h_{ref}}^{z_2=h} \frac{z - h_P}{r^3}\, dzdydx\,, \; r = \sqrt{(x - x_P)^2 + (y - y_P)^2 + (z - h_P)^2} \tag{1}$$

where k is the Newton's gravitational constant, ρ is an average density of the topographic mass (=2.67 g/cm^3), and h are the topographic heights from, given by, for instance, SRTM. If the mean elevation surface is a sufficiently long wavelength, then we can approximate Eq. (1) as

$$\delta g_{RTM}(P) \approx 2\pi k\rho(h_P - h_{ref}) - k\rho \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{z_1=h_P}^{z_2=h} \frac{z - h_P}{r^3}\, dzdydx \tag{2}$$

It should be noted that the first term of Eq. (2) refers to two Bouguer plates at h_P and h_{ref}, and the second one refers to terrain correction. In practice, $\delta g_{RTM}(P)$ is numerically computed by summation of N rectangular prisms within some radius around the evaluation point (P). As shown in Figure 1, the single prism is defined by the coordinates x_1, x_2, y_1, y_2, z_1, and z_2 in the left-handed coordinate system. Therefore, the second term of Eq. (2) can be written in the closed analytical forms of flat-top prisms as follows Sansó and Rummel (1997):

$$\delta g_{RTM}(P) \approx 2\pi k\rho(h_P - h_{ref}) - \sum_{i=1}^{N}\left(k\rho \,\|\|\, x\ln(y + r) + y\ln(x + r) - z\arctan\frac{xy}{zr}\Big|_{x_1}^{x_2}\Big|_{y_1}^{y_2}\Big|_{h_P}^{h}\right)_i \tag{3}$$

In fact, the summation term of Eq. (3) is terrain corrections. Our aim is to assess gravity requirements with respect to spatial resolutions. We assume that, in the void areas, EGM2008 contributes long- and medium-wavelength information of the earth's gravity field. For all wavelength contents, the topography-implied gravity anomalies in those areas can be approximated by EGM2008 gravity anomalies, Δg_M, and δg_{RTM} as follows:

$$\Delta g \approx \Delta g_M - \delta g_{RTM} - 2\pi k\rho h_{ref} \tag{4}$$

In Eq. (4), refined Bouguer gravity anomalies [complete Bouguer gravity anomalies plus terrain corrections (Heiskanen and Moritz, 1967, p. 131)] are immediately obvious if we consider Δg_M as free-air gravity anomalies. For numerical computations with an average topographic mass (crust) density of 2670 kg/m^3, we calculated δg_{RTM} using 30 arcsecond SRTM data [derived from three-arcsecond SRTM data--averages over 30 arcsecond × 30arcsecond blocks] and stored it in database—the term h_{ref} were generated using DTM2006.0. The simulated amomalies in Eq. (4), then, were computed.

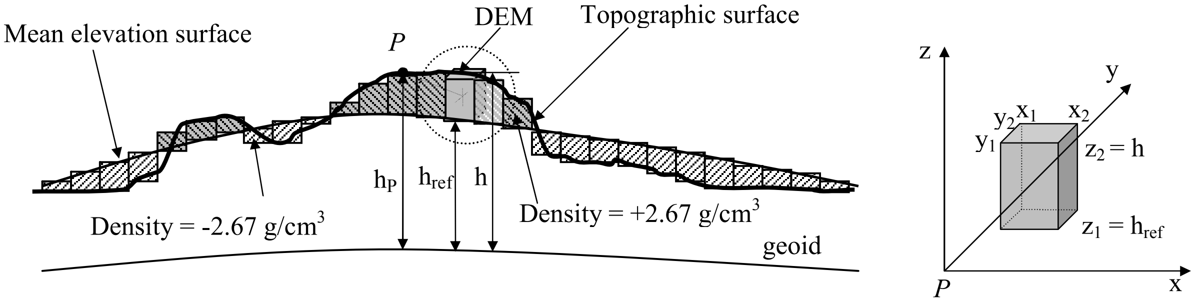
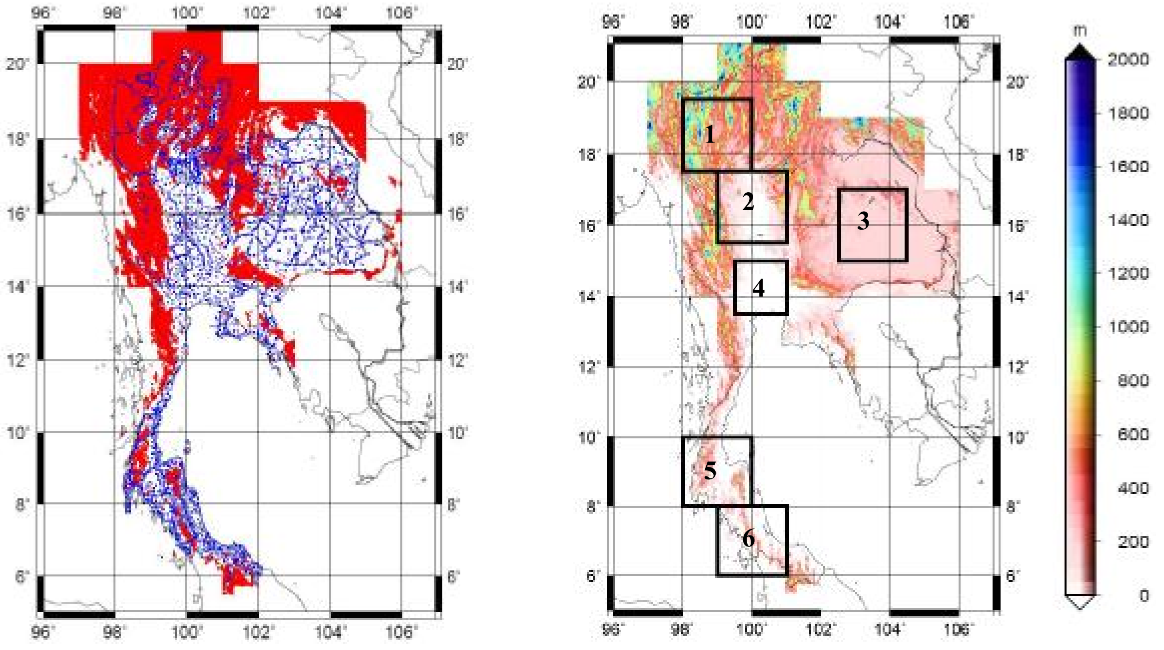


Figure 1: The residual terrain model



(a)

(b)

Figure 2: (a) Locations of existing terrestrial gravities (blue rectangular dots) and higher 400-m elevations (red rectangular dots); (b) Topography of six testing areas (black rectangles)

Figure 2 depicts 3,979 terrestrial gravity stations, provided by Royal Thai Survey Department (RTSD) and topography in Thailand. However, only 3,949 stations were employed for geoid computations as the other 30 stations remain questionable (see Figure 2a). These stations are referred to the International Gravity Standardization Net 1971 (IGSN71). Some parts of the region, e.g., northwest, southwest, and east areas, are mountainous and inaccessible. Obviously, the resolutions in those areas are not uniform, with data mostly following existing roads, and large data gaps are in the order of over 50 km (27.8 arcminutes). In the other areas, the distributions of data are more uniform, and their resolutions are roughly 2-10km (1.1 – 5.5 arcminutes). Six testing areas, where existing data are too scarce, were chosen according to their smoothness and roughness, as shown by black rectangles in Figure 2b Table 1 lists their boundaries and statistic details.

We consider filling the topography-implied (or simulated) anomalies in the areas with higher 400-m elevations because of not only less correlation (linear relationship) of RTSD free-air anomalies with respect to lower elevations (not shown in this work) but also the number of data and the distribution of these data, similar to the case study of geoid computation in the Malaysian peninsula as stated in Vella (2003). However, lower elevations could be significant, but are not considered in this work. In this study, we simply assume all types of gravimetric quantities are consistent. In fact, this can cause a dm-error level in geoid computation, and further study will be needed. The augmentation of the existing RTSD gravity data with the topography-implied gravity anomalies were shown in Figure 2a. We use EGM2008-only to mitigate the edge effects in the geoid computation due to no gravity data available in ocean areas and land areas outside the Thailand territory.

Table 1: Statistic details of six testing areas; units of meters

Area		Min	Max	Mean	Std.	rms
1	17.5ºN< φ <19.5ºN and 98.0ºE < λ <100.0ºE	57	2522	655	339	738
2	15.5ºN< φ <17.5ºN and 99.0ºE < λ <101.0ºE	20	2000	211	277	348
3	15.0ºN< φ <17.0ºN and 102.5ºE < λ <104.5ºE	115	631	173	53	181
4	13.5ºN< φ <15.0ºN and 99.5ºE < λ <101.0ºE	0	538	30	51	59
5	8.0ºN< φ <10.0ºN and 98.0ºE < λ <100.0ºE	0	1548	96	171	196
6	6.0ºN< φ <8.0ºN and 99.0ºE < λ <101.0ºE	0	1196	49	107	118

GEOID COMPUTATION AND DATA REQUIREMENTS

In this study, the geoid undulation N is computed through the generalized Stokes' integral (Heiskanen and Moritz 1967). All computations are in the non-tidal system. With the usual remove-and-restore procedure, the geoid undulation is defined as follows:

$$N = N_M + \frac{R}{4\pi\gamma}\iint_{\sigma}(\Delta g_F - \Delta g_M)S(\psi)d\sigma \qquad (5)$$

where σ is the area of integration, R is the mean radius of the Earth, γ is normal gravity on WGS84 ellipsoid (Somigliana's formula in Heiskanen and Moritz (1967, p. 70)), $S(.)$ is Stokes' function with spherical distance ψ, and Δg_F is the free-air gravity anomaly with terrain correction (called Faye anomaly , used to approximate Helmert gravity anomaly). The symbols "Δg_M" and "N_M" are the gravity anomaly and the geoid undulation, generated by EGM2008 at degree 2 to 2190, respectively; more details can be found in Pavlis et al (ibid). For this study, we neglect indirect effect (Wichienchareon 1982). One dimensional (1-D) spherical Fast Fourier Transform (FFT) of Haagmans et al. (1993) was used to evaluate Stokes' integral in Eq. (5), which require gridded data.

The data resolutions required to estimate omission errors in geoid undulation can be determined by numerical comparisons of N with different levels of data spacing. We prepared Δg_F gridded data on $30''\times30''$, $1'\times1'$, $2'\times2'$, and $3'\times3'$ grids. The corresponding grids of Δg_F were interpolated from the scatteredly measured points using a method of continuous curvature spines in tension in the Generic Mapping Tools (GMT) (Smith and Wessel 1990; Wessel 2009). The tension factor of $T = 0.75$ was selected to minimize the impact of gravity errors in mountainous areas on adjacent grid points without gravity data as suggested by Smith and Milbert (1999).

The residual geoid undulations on regular grids were computed, according to the second term of Eq. (5) using 1-D spherical FFT in six testing areas (see Figure 2b). The FFT was conducted on the residual grids, $\Delta g_F - \Delta g_M$ using 100% zero-padding on the east and west edges of the grid to eliminate the effect of cyclic convolution. The geoid undulations were obtained by the restoration of N_M. For numerical comparisons, we used N on $30''\times30''$ grid as true geoid undulations, symbolized by N_{true}, and others were the estimated undulations, N_{est}.

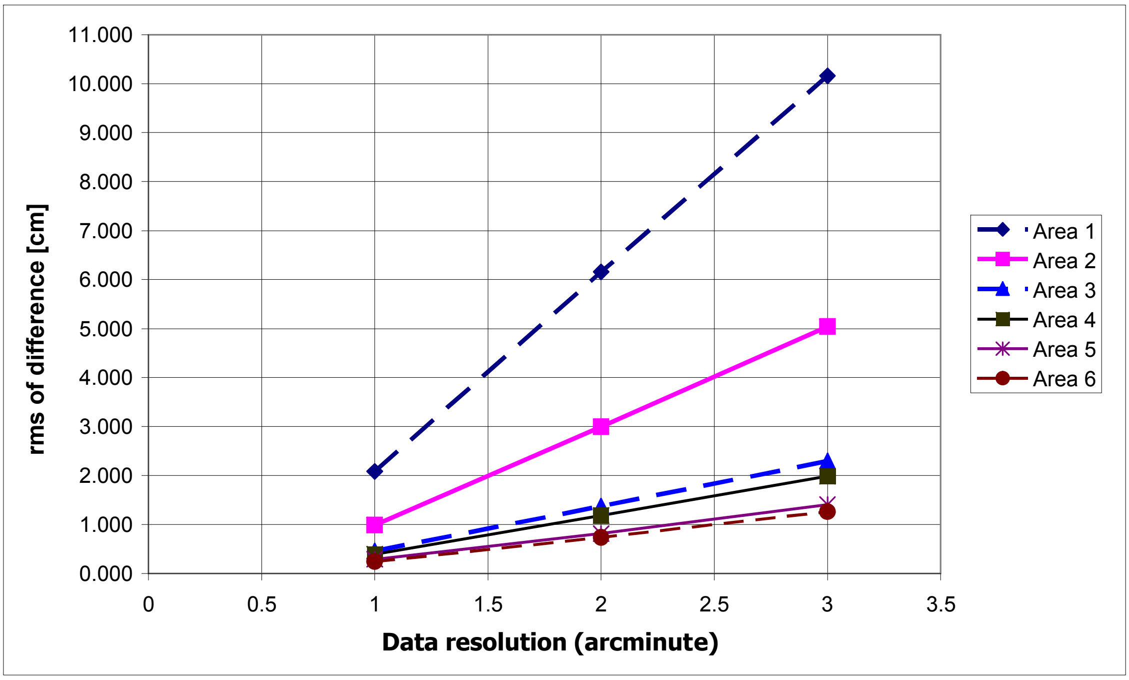


Figure 3: The relationship between root-mean-square (rms) of differences, $N_{est} - N_{true}$, and data resolutions

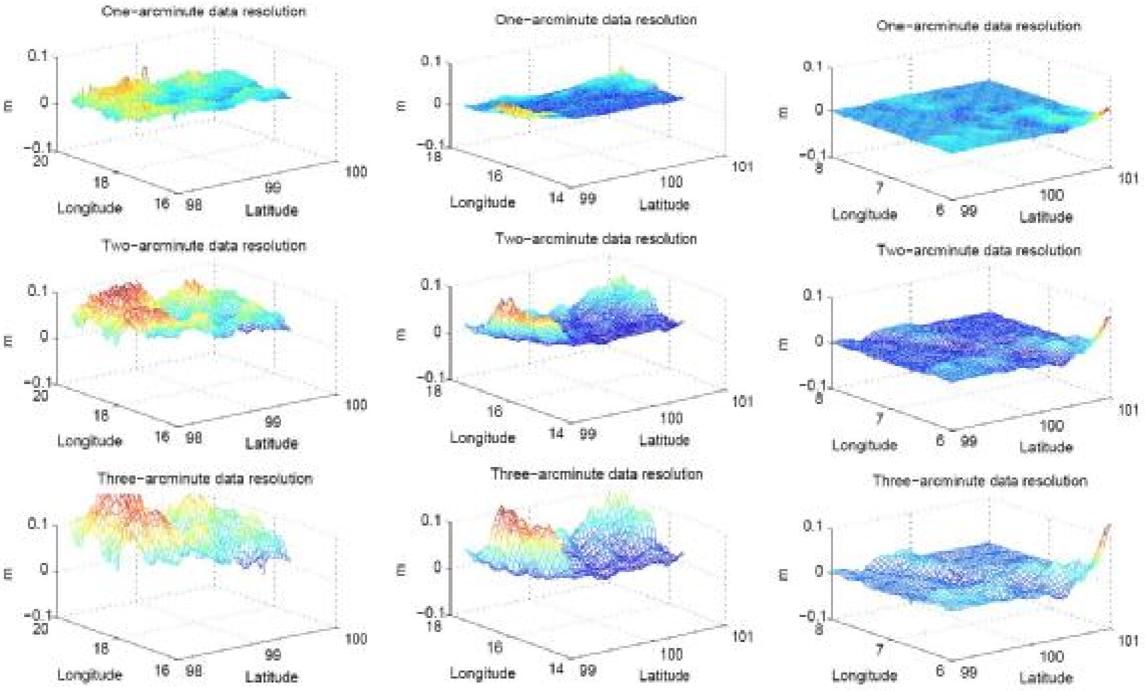
According to Jekeli et al. (2009), if the commission and omission errors should contribute equally, then a total root-mean-square (rms) error of 5cm would require an omission rms error of 3.5 cm. Thus, from Figure 3, in mountainous area (area 1), the data resolution should be about 1.47′ (2.6 km). In other words, the omission error-can grow to 10 cm if data resolution is about 3′ (5.5 km); the errors decrease to a few cm if data resolution is about 1′ (1.8 km). For the smoother areas, 3, 4, 5, and 6, the omission errors are less than 1 cm at data resolutions of about 1′. Table 2 provides statistics of differences, $N_{est} - N_{true}$, in association with data resolutions. Figure 4 shows the errors of, for instance, rough area 1, moderate area 2, and flat area 6, the isolated peaks in errors become larger, due to lower data resolution.

CONCLUSIONS & RECOMMENDATIONS

This study provides the assessment of gravity requirements for geoid determination at certain levels of errors in Thailand. Six testing areas with different mean elevations were chosen. We generated topography-implied gravity anomalies using three-arcsecond SRTM and EGM2008 by means of RTM, according to Eq. (4) over those areas. With the assumption of data consistency, the anomalies were simply augmented to existing RTSD gravity data, yielding high resolution gravity anomalies. The geoid computations were computed on 30″×30″, 1′×1′, 2′×2′, and 3′×3′ grids. Using these numerical computations, we showed that rough areas, for instance, require more gravimetric data intensities (or higher spatial resolutions) to meet specific geoid accuracy requirements, such as 5 – 10 cm (rms) for 3″×3″ grid. However, for flat areas, we equally achieve a certain accuracy level (1-2-cm rms) with data resolutions of one to three arcminutes. However, in this study, we have not yet considered commission errors due to observation noises related to data resolution, which can cause geoid errors up to a few centimeters with 2′×2′ grid resolution and 5mGal noises included as suggested by Jekeli et al (2009) where South Korea areas were tested, further analyses will be needed for precise geoid determination in Thailand.

Table 2: Statistics of differences, $N_{est} - N_{true}$, due to limited data resolution; units of cm

Area	Resolution	Min	Max	Mean	Std.	rms
1	1'×1'	−2.010	6.870	1.847	0.969	2.085
	2'×2'	−1.970	12.450	5.511	2.747	6.158
	3'×3'	−4.000	20.900	9.097	4.518	10.156
2	1'×1'	−0.900	5.190	0.712	0.690	0.991
	2'×2'	−0.600	11.530	2.164	2.070	2.995
	3'×3'	−1.100	17.500	3.636	3.494	5.042
3	1'×1'	−0.300	1.210	0.428	0.170	0.461
	2'×2'	−0.690	3.410	1.286	0.494	1.377
	3'×3'	−1.000	5.600	2.141	0.826	2.295
4	1'×1'	−0.200	1.650	0.331	0.215	0.395
	2'×2'	−0.400	4.850	0.995	0.645	1.186
	3'×3'	−0.700	7.700	1.665	1.084	1.987
5	1'×1'	−2.180	1.820	0.042	0.289	0.292
	2'×2'	−4.570	2.330	0.130	0.807	0.817
	3'×3'	−8.100	3.800	0.189	1.392	1.405
6	1'×1'	−1.050	2.910	0.094	0.228	0.246
	2'×2'	−1.580	8.070	0.289	0.678	0.737
	3'×3'	−3.700	13.200	0.469	1.174	1.263



(a) (b) (c)

Figure 4: Differences, $N_{est} - N_{true}$, for the case of (1) area1(rough), (b) area 2(moderate), and (c) area 6(flat)

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