NUMERICAL APPROACH FOR ANALYZING CRITICAL CONFIGURATIONS OF SINGLE PHOTO RESECTION

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ABSTRACT: Single photo resection for estimating exterior orientation parameters can be achieved by using control points, control lines, or effective control data of other types. However, orientation solution would become largely uncertain or even impossible to solve when faced with poor layout of control data. This study mainly focused on identifying such geometrically configurations when employing point features and line features. The authors utilized rank analysis together with condition number to spatially locate the area of control points and control lines that fall into singular solutions. Consequently, effective observations toward a satisfactory solution can be then arranged.

1. INTRODUCTION

Geometric analysis of single photo resection is one of the significant tasks often seen in photogrammetry and computer vision related applications. Grunert (1841) utilized three control points to solve single photo resection in a linear mathematical model, and a step further to analyze critical configurations which might incur singularity under the condition of minimum solution. Since then, studies focused on the critical configurations of control points solving for single photo resection continued and the major discoveries can be summarized as below:(1) Collinear layout(Gotthardt, 1940) and danger circle, (2) danger cylinder (Wunderlich, 1943) and (3) horopter curve (Killian, 1990). On the other hand, the employment of control lines have recently gotten attention in photo orientation missions. Conceptually, the so-called interpretation plane that passes the perspective center, the image line segment, and the associated line segment in the object space can be utilized for analyzing if the involved interpretation planes would suffice to uniquely determine the perspective center or not. It was analytically derived in Perng (2005) that at least three control lines causing no singularity are needed for solving exterior orientation parameters of a single photo.
To analyze the critical configurations, there were different approaches found effective. Faig(1986) utilized condition number to investigate stability of the solution system. This approach was based on linear mathematical model. In this study, the critical configurations of both point-based and line-based single photo resections based on collinearity equations were to explored. The methods considered are given in the section that follows.

2. METHODS

Collinearity equations is utilized as mathematical model in this study, shown in Eq.(1) shows the collinearty equations. It is easy to realize that three control point measurements would provide six equations that would exactly balance with six exterior orientation parameters.

$$\begin{aligned} F_{2n-1} &= -f\,\frac{\left[m_{11}(X_n-X_L)+m_{12}(Y_n-Y_L)+m_{13}(Z_n-Z_L)\right]}{\left[m_{31}(X_n-X_L)+m_{32}(Y_n-Y_L)+m_{33}(Z_n-Z_L)\right]}-x_n+x_0 \\ F_{2n} &= -f\,\frac{\left[m_{21}(X_n-X_L)+m_{22}(Y_n-Y_L)+m_{23}(Z_n-Z_L)\right]}{\left[m_{31}(X_n-X_L)+m_{32}(Y_n-Y_L)+m_{33}(Z_n-Z_L)\right]}-y_n+y_0 \end{aligned} \tag{1}$$

where n is the number of control point; (x_n , y_n)are image coordinates, x_0 , y_0 , and f are interior orientation parameters; ($m_{11}, m_{12}, \ldots m_{32}, m_{33}$) are the elements of rotation matrix; (X_n, Y_n, Z_n) are the object coordinates of control point; (X_L, Y_L, Z_L) are the object coordinates of perspective center.

Replacing object coordinates in Eq. (1) with line parametric equation (Mulawa and Mikhail, 1988), shown in Eq. (2), Line-based collinearity equations would be formed accordingly. The t parameters in line-based collinearity equations are obviously the unknowns left to be solved. It turns out that three control line segment measurements, two end points for each line segment, would be minimally required for single photo resection.

$$\begin{bmatrix} X_n \\ Y_n \\ Z_n \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} X_1-X_0 \\ Y_1-Y_0 \\ Z_1-Z_0 \end{bmatrix} t = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} dX \\ dY \\ dZ \end{bmatrix} t \tag{2}$$

where X_0, Y_0, Z_0 , X_1, Y_1, Z_1 are two points of one line; dX , dY , dZ are direction vector; t is parameter of line.

Jacobian matrix is the matrix of the partial derivatives of collinearity equations. Based on points or lines, are separately corresponding in Eq. (3). Since equations are non-linear, an iteration-computed should be required. In the first step, substituting initial value into incremental of unknowns shown in Eq. (4), adjustmenting to renew initial values and find out best estimate values of the unknowns. Normal matrix which denoted as *N*, if *N* nears to singularity, the inverse of N should not be computed. There are two main factors impact the structure of *N*: distributions of control data and its accuracy. However, how the distribution of control data influence on the stability of system is the main idea of this study. Therefore, iteration and initial values problem are neglected.

$$A=\frac{\partial F}{\partial_{\text{unknown}}}\Big|_{X_0,l_0}=\begin{bmatrix}\frac{\partial F_1}{\partial X_L} & \frac{\partial F_1}{\partial Y_L} & \frac{\partial F_1}{\partial Z_L} & \frac{\partial F_1}{\partial \omega} & \frac{\partial F_1}{\partial \varphi} & \frac{\partial F_1}{\partial \kappa}\\ \frac{\partial F_2}{\partial X_L} & \frac{\partial F_2}{\partial Y_L} & \frac{\partial F_2}{\partial Z_L} & \frac{\partial F_2}{\partial \omega} & \frac{\partial F_2}{\partial \varphi} & \frac{\partial F_2}{\partial \kappa}\\ & & \vdots & & & \\ \frac{\partial F_6}{\partial X_L} & \frac{\partial F_6}{\partial Y_L} & \frac{\partial F_6}{\partial Z_L} & \frac{\partial F_6}{\partial \omega} & \frac{\partial F_6}{\partial \varphi} & \frac{\partial F_6}{\partial \kappa}\end{bmatrix}_{6\times 6}=\begin{bmatrix}\frac{\partial F_1}{\partial X_L} & \frac{\partial F_1}{\partial Y_L} & \frac{\partial F_1}{\partial Z_L} & \frac{\partial F_1}{\partial \omega} & \frac{\partial F_1}{\partial \varphi} & \frac{\partial F_1}{\partial \kappa} & \frac{\partial F_1}{\partial t_{1\_line1}} & \frac{\partial F_1}{\partial t_{2\_line1}} & \cdots\frac{\partial F_1}{\partial t_{2\_line3}}\\ \frac{\partial F_2}{\partial X_L} & \frac{\partial F_2}{\partial Y_L} & \frac{\partial F_2}{\partial Z_L} & \frac{\partial F_2}{\partial \omega} & \frac{\partial F_2}{\partial \varphi} & \frac{\partial F_2}{\partial \kappa} & \frac{\partial F_2}{\partial t_{1\_line1}} & \frac{\partial F_2}{\partial t_{2\_line1}} & \cdots\frac{\partial F_2}{\partial t_{2\_line3}}\\ & & & & \vdots & & & & \\ \frac{\partial F_{12}}{\partial X_L} & \frac{\partial F_{12}}{\partial Y_L} & \frac{\partial F_{12}}{\partial Z_L} & \frac{\partial F_{12}}{\partial \omega} & \frac{\partial F_{12}}{\partial \varphi} & \frac{\partial F_{12}}{\partial \kappa} & \frac{\partial F_{12}}{\partial t_{1\_line1}} & \frac{\partial F_{12}}{\partial t_{2\_line1}} & \cdots\frac{\partial F_{12}}{\partial t_{2\_line3}}\end{bmatrix}_{12\times 12} \tag{3}$$

where F_1, F_2 ..., F_{12} are equations; X_L, Y_L, Z_L, ω, φ, κ, t_{1_line1}, t_{2_line1} ..., t_{2_line3} are unknown parameters; X_0 are initial values of unknown parameters; l_0 are coordinates of control data and its corresponding points on image.

$$\hat{\xi}=(A^T PA)^{-1}A^T Pw=N^{-1}A^T Pw \tag{4}$$

Where $\hat{\xi}$ is incremental of unknowns; *A* is jacobian matrix; *P* is weight matrix, *w* is discrepancy vector.

The stability of solution system could be evaluated by condition number of *N*. If results are largely influenced by small error, condition number consequence will be larger, then the solution system is so-called ill-conditioned. Condition number is defined by norm of *N*, also by eigen-decomposed, the absolute value of maxmum-eiginvalue divides to minmum-eiginvalue, shown in Eq. (5).

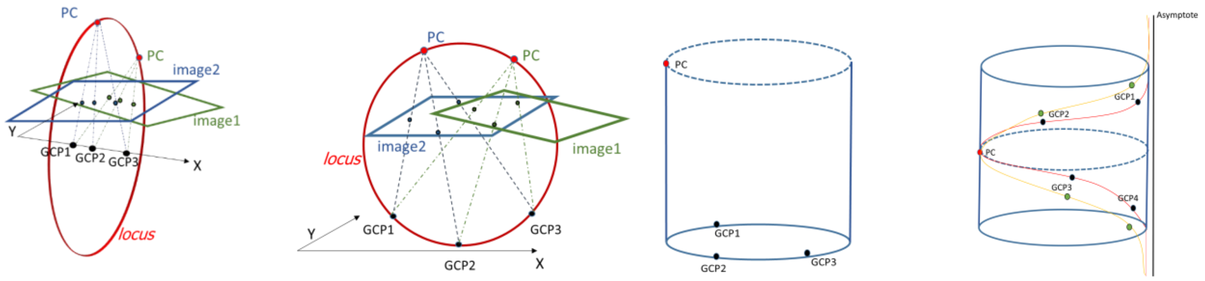
$$c(N)=\|N\|\cdot\|N^{-1}\|=\left|\frac{\lambda_{\max}}{\lambda_{\min}}\right| \tag{5}$$

where $\|N\|$ is the norm of normal matrix; $\lambda_{\max}$ is the maximum of eigenvalue; $\lambda_{\min}$ is the minimum of eigenvalue.

For control point, the critical configurations are collinear layout, danger circle, danger cylinder and horopter curve. For control line, the critical configurations are parallel and intersect at a point between at least three lines. The characteristic of these six configurations will be discussed in the following sections.

2.1 Control Point

Collinear layout under the condition of minimum solution as shown in figure 1(a), perspective center wll locate on circumference in geometry. In figure 1(b), when three control points together with perspective center are on a circle, perspective center will be anywhere on circumference since circumferential angles are the same. In figure 1(c), three control points and perspective center are on the surface of a cylinder. In such case, however, not anywhere in the surface be the correct position of perspective center, only three positions are possible(Zhang, 2006). *N* will incur singularity in all three cases afore-mentioned in numerical approach. In binocular vision, there exist special curve that any point on the curve is imaged on two retinas yield single vision. Such curve called horopter curve formed by four distinct points. It can also be written in closed form, in Eq. (6) (Killian, 1990). However, critical configuration of horopter curve in single photo resection isn't defined exactly the same as relative orientation. Figure 1(d) shows this kind of critical configuration.



(a) (b) (c) (d)

Figure 1. Critical configurations of control points. (a) collinear layout (b) danger circle (c) danger cylinder (d) horopter curve (modified from Tilch, 2012)

$$Z_n = \frac{k\sqrt{X_n}}{\sqrt{a - X_n}} \;;\; Y_n = \sqrt{X_n\left(a - X_n\right)} \;;\; 0 \leq X_n \leq a \tag{6}$$

Where k is the parameter of the horopter curve; a is the diameter of the circle; (X_n , Y_n , Z_n)are object coordinates of control points.

2.2 Control Line

Generally, *N* will suffer singular when three control lines are parallel or intersect at a point, as shown in figure 2(a) and figure 2(b) separately. Geometrically, control lines and a given perspective center would form the interpretation planes. These three planes intersect at one line, therefore the possible position of perspective center would locate on specific intersect line.

(a) (b)

Figure 2. Critical configurations of control lines. (a) parallelism (b) intersect at a point

3. EXPERIMENTS

All data bellows were simulated based on aerial photo case, given an assumption that focal length is 0.2m, principal distance are zeros, flying height is 2000m. The distribution of control data is the only independent variable to be considered in this study, therefore orient parameters are all set in 0 degrees.
To analyze critical configurations in both of control point and of control line, rank analysis is numerical tool to judge whether *N* is singular or not. Stability of *N* can be indicated by condition number.
For control point, as mentioned in chapter 2.1, perspective center might locate everywhere on specific locus, three angles are formed by each possible perspective center together with three control points. If the three angles are the same when perspective center is anywhere on such locus, then it can make sure that the locus is exactly the possible position on the condition of unstable solution systems.
Unlike the critical configurations of control point, there are no specific locus of perspective centers for control line. Instead, the intersection of three interpretation planes play an important role on the accuracy of perspective center. Therefore, the angles of the normal vectors of three interpretation planes and the angles of cross vectors of normal vectors will be the indicator of intersecting geometry.

3.1 Critical configuration of control point

Fixing the position of two control points GCP1 and GCP2 in figure 3, the third control point are arranged in a 2300m*2300m ground coverage with elevation of all points are equal to zeros, generating totally 47*47 geometric configurations. When the control points are on a straight line or control points together with perspective center which are on a circle in X-Y plane, condition number of *N* is much larger, where critical configuration happens and results in poor stability of solution systems.

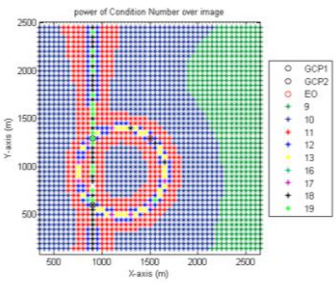


Figure 3. Condition number in 2300m*2300m ground coverage.

In figure 4(a), when control points meet collinear layout, solution systems have one rank-deficient thus causing singular. In geometry, the locus of perspective center would be a circle, anywhere on the locus all satisfy the invariance value of the three angles. In numerical approach, *N* must not be inversed because lack of geometry information. As for condition number, the value is about 10 thousand times than regular configurations. The critical configuration of danger circle is shown in figure 4(b). If control points and perspective center are on the same circle, rank-deficient would happen. And anywhere on the circle is possible, except for intervals that perspective center can't form circumferential angle with control points. Generally, the value of condition number is about one million times than regular configurations. Danger cylinder, shown in figure 4(c), a circle formed by three control points on X-Y plane, when perspective center locates on the surface of a cylinder, *N* would definitely encounter singularity. However, almost all perspective centers on cylinder could not conform the same angles which formed with control points. Under that circumstances, it tells the truth that correct location of perspective centers are actually limit to a few. Furthermore, the larger scale doesn't make solution system more stable. Final comes to horopter curve, shown in figure 4(d), when control points and perspective center are on the same horopter curve, especially perspective center locates on the other side of midpoint of asymptotic line. Under such configuration, *N* will also encounter singularity.

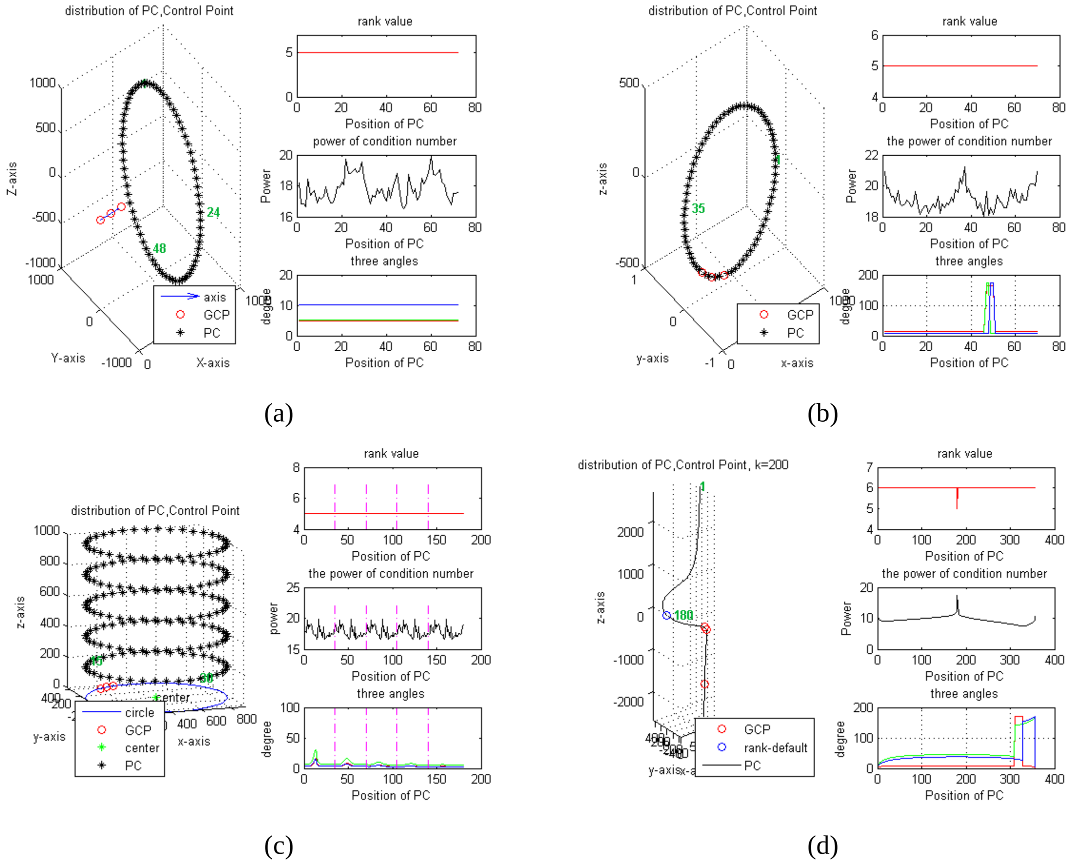
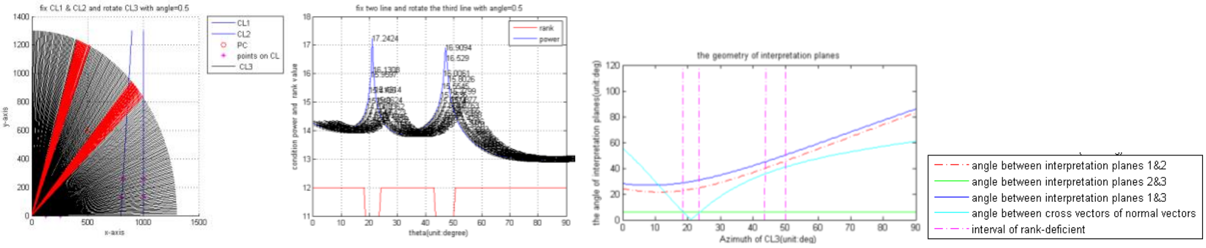


Figure 4. Critical configuration of control point. (a) collinear (b) danger circle (c) danger cylinder (d) horopter curve

3.2 Critical configuration of control line

Fixing two control lines CL1 and CL2 in figure 5(a), the third control line are arranged from zero to ninety azimuth, generating totally 180 geometric configurations. When CL3 is marked with red, then such configuration is numerically singular. In figure 5(b), condition number is computed while each configuration formed. There are two local maximum which corresponding two critical configurations: three extended of control lines are coincide at a point, the other critical configuration is then unconventional. Corresponding these two local maximum to figure 5(c) which shows the geometry of interpretation planes, angle between cross vectors of normal vectors is zero and non-zero separately. Value zero means that all the three normal vectors are coplanar, thus, two critical configurations of intersecting at a point or parallelism are consequence the critical configurations. Value non-zero, however, is not provide with such property, the corresponding critical configuration reveals poor intersection of three planes as figure 6 shown, the accuracy of intersect-point is not so well. In regards to facts mentioned above, set the fixing two control points have larger intersect angle in figure 7(a) shown, the geometry of interpretation planes in figure 7(c) are better than figure 5(c)shows. Therefore only the case that three extended lines are coincide at a point remains. Therefore, the geometry of three interpretation planes is crucial. When it comes to three identical interpretation planes but with different orientation or translation control line on it, configurations are shown in figure 8(a) (d) (g), the forming interpretation planes are all the same as figure 2(a). So the geometry of itself in consequence the same as figure 8 (c) (f) (i) shown. As to stability of *N* shown in figure 8 (b) (e) (h), figure 8 (b) is apparently different from the others. Unconventional critical configuration shows up and condition number generally much larger. So, it tells that when control lines are coplanar, the worse the stability of solution system.



(a) (b) (c)

Figure 5. Geometric configurations. (a) configuration (b)condition number and rank value(c)geometry of planes.

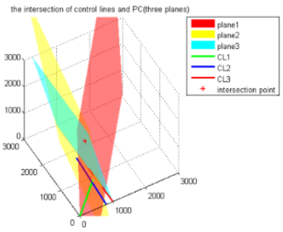
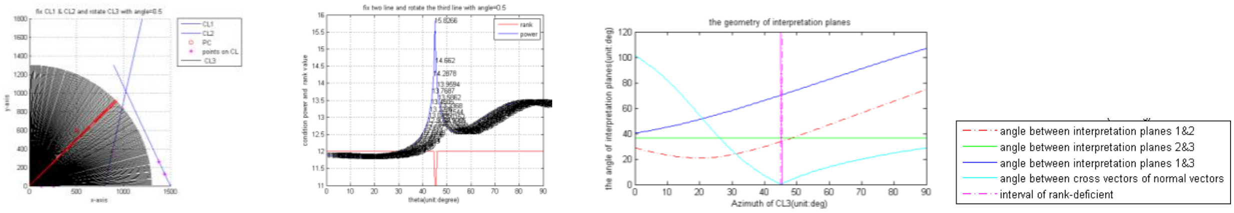
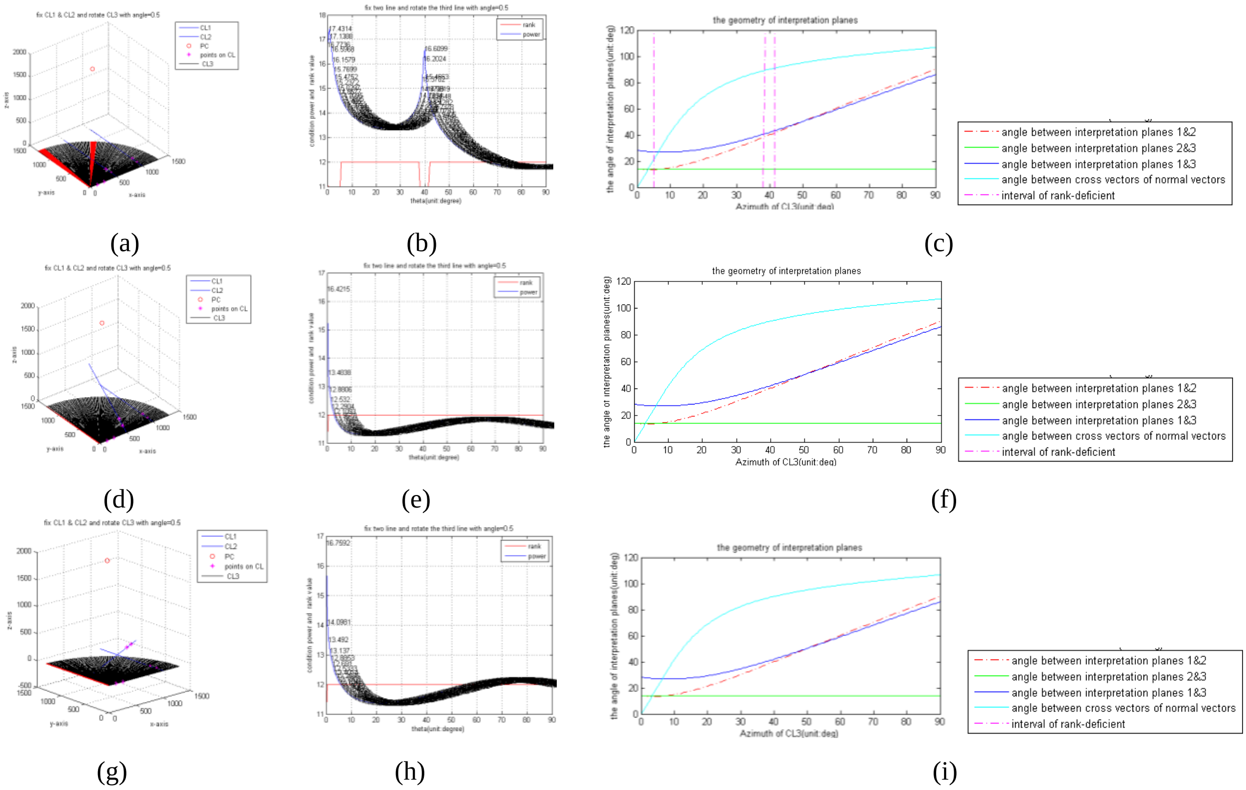


Figure 6. Intersection of planes under unconventional critical configuration.



(a) (b) (c)

Figure 7. Geometric configurations. (a)configuration (b)condition number and rank value(c)geometry of planes.



(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Figure 8. (a) (d) (g) configrations (b) (e) (h) condition number and rank value (c) (f) (i) geometry of planes.

As table 1 shown, points or lines are utilized as control data, normal matrix could probably incur singularity when specific geometric configuration formed. For point features, three control points are collinear or on the circle lead to singularity due to its geometric-insufficient; when three points together with perspective center are on cylinder surface, due to special phenomenon that points on such critical surface can lead to numerical instability; when three control points and perspective center are on horopter curve, is most special case because singularity could not always happen. But it is for sure that when perspective center is exactly on the other side of midpoint of asymptotic, instability of solution system will be happen. For line features, when three lines are parallelism or intersect at a point, due to its geometric-insufficient, will similarly incur singularity; when lines are on unconventional critical configuration, solution system can also face with singularity due to its poor intersect accuracy.

In both point and line features, singularity happened due to geometric-insufficient, there exist specific locus for perspective center; if control data and perspective center are distribute on critical surface, the probable locations of perspective center will limit to a few. When it comes to regular configurations which have sufficient geometric information, perspective center is uniquely to be determined. Adjusting geometric configurations of control data base on minimum number, if singularity happened, it is always lead to one rank-deficient number. Finally, compare with critical configurations and regular configurations, the ratio of condition number is at least 1 and ups to 10^3.

Table 1. Configurations of control data.

Control data	Normal matrix	Causes	Uniqueness of P.C	Ratio of Condition Number(**)	Rank-deficient number	Correspond configurations
Points	Singular	geometric-insufficient	non-unique	10^3	1	collinear danger circle
		distribute on critical surface	Ambiguity(*)			Danger cylinder
		distribute on critical surface	ambiguity			Parts of horopter curve
	Non-singular	geometric-sufficient	unique	1	0	Most of horopter curve and regular geometry
Lines	Singular	geometric-insufficient	non-unique	10^3	1	parallelism and intersect at a point
		poor intersect accuracy	unique	1 to 10^3	1	unconventional critical configuration
	Non-singular	geometric-sufficient	unique	1	0	regular geometry

(*) Ambiguity means there are different number of solutions in regards to relation between P.C. and control points.

(**)Ratio of condition number defined as condition number divided by singular and non-singular matrix. In this study, condition number of non-singular matrix both in points or lines are set to 1.

4. CONCLUSION

In this study, both critical configurations of control point and control line based on single photo resection have been identified, was already presented in table 1. Through the study, user can arrange better geometric configuration to avoid numerical singularity. Besides of that, since rank-deficient number is one, it tells the truth that there are losing one necessary information. Thus, if identify undetermined parameter that leads N to singular could be possible, adding efficient observations into solution system could reach full-rank, Finally, solving unknown parameter would be possible.

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