

# SPATIAL SCALING TRANSFORMATION MODELING OF VEGETATION LEAF AREA INDEX RETRIEVED BY REMOTE SENSING IMAGE BASED ON FRACTAL

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**KEY WORDS:** scaling transfer model; fractal dimension; standard deviation

## ABSTRACT

Scaling transformation is one of the basic and important scientific questions in quantitative remote sensing. This study proposed a leaf area index (LAI) scaling transfer model based on fractal theory for computing LAIs at different scales (spatial resolution). Based on scale invariance and self-similarity of remote sensing images in a statistical sense, the LAI scaling transfer model was developed by establishing the double logarithmic linear relationship between the scale  $n$  and the average LAIs of the image at different scales. The influence of the standard deviation of the image on the scaling transfer model was also analyzed. The results showed that the average LAIs of the image at different scales were well calculated by the scaling transfer model with an absolute percent error (APE) value of 0.27% and a root mean square error (RMSE) value of 0.0129. The fractal dimension of the image, the parameter of the scaling transfer model, increased as the standard deviation increased. This study suggests that the proposed method of LAI spatial scaling transformation based on fractal theory is feasible.

## 1. INTRODUCTION

Scale issue is not only associated with these subjects, such as mathematics, computer science, signal processing and ecology, but also one of the most essential and difficult questions in the field of remote sensing (Silvestri et al., 2002). As one of the important surface parameters that can be inverted using remote sensing data, the leaf area index (LAI) is a key player within a broad range of land surface models including vegetation, biogeochemical or global atmospheric circulation models, which necessitate the improvement and assessment of the accuracy of LAI estimation. The scale effect of LAI inversion using remote sensing has been extensively examined, focusing on describing the phenomenon, analyzing the causes and establishing the scale transformation relationships. Establishing the transformation relationship among the inversion LAIs at different scales, namely scale transformation, is an effective method that resolves the scale effect in quantitative remote sensing. For quantitative description of scale transformation of inversion LAI, the linear or non-linear transformation relationships among LAIs at different scales were established with statistical analytical methods (Liang et al., 2002; Garrigues et al., 2006; Jin et al., 2007). However, a large number of sample data are needed and there is no clear physical significance of the parameters in these models, leading to the limited applicability of the transformation. Therefore, with the advancement of quantitative remote sensing theory, some scholars used physical models to deduce the LAI scale transformation law by analyzing the biophysical mechanism associated with the scale effect (Xu et al., 2009). Mathematical methods are also used to study the spatial scale transformation of LAI, besides statistical analytical methods and mechanism models. As one of classical methods of scale transformation, fractal can quantitatively describe the evolution law of study objects at different scales (Luan et al., 2013). Li et al (1999) proposed that fractal (or similar fractal) relationship is one of the three types of scale transformation tendency that can reflect the scale effect when physical law, theorem and model are applied in remote sensing. This paper

proposed a fractal-dimension-based LAI spatial scale transfer model for the calculation of and transformation among LAIs at different scales.

## 2. MATERIALS AND METHODS

### 2.1. Materials

The study area is located in the suburb of Changchun city, Jilin Province, China, which is situated in the Northeastern Plain and the terrain is rather flat (Fig. 1). The CCD image (the spatial resolution: 30m) obtained from the Environment and Disaster Reduction Small Satellites was adopted in this paper. The study area is mainly covered by farmland and sparsely distributed buildings, roads, water bodies and such.

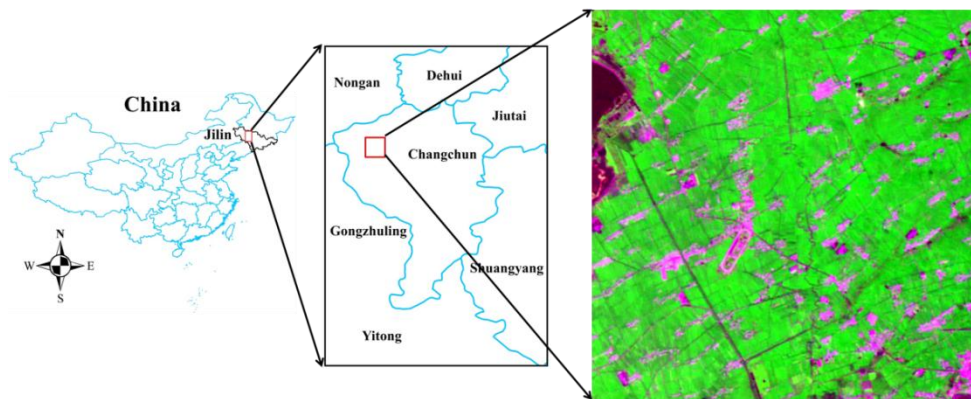


Figure 1 Location and CCD image of the study area in Changchun, Jilin Province, China.

This paper used Normalized Difference Vegetation Index (NDVI) to invert LAI. An empirical transfer function  $f$  (Eq. (1)) is established using the NDVIs derived from the CCD image and the measured LAIs to generate the LAI map using the CCD image.

$$LAI = 0.2258e^{3.727NDVI} \quad (1)$$

### 2.2. Methods

There are two up-scaling methods of LAI inversion. The exact value of the LAI ( $LAI^{exa}$ ) at the coarser spatial resolution is obtained by first applying  $f$  to the vegetation index ( $VI_i$ ) at the high spatial resolution to calculate the corresponding LAI value ( $LAI_i$ ) and then by aggregating the results of  $LAI_i$  (*path 1*). The approximated value of the LAI ( $LAI^{app}$ ) at the coarser spatial resolution is obtained by first aggregating the result of  $VI_i$  ( $VI_m$ ) at the high spatial resolution and then by applying  $f$  to  $VI_m$  at the coarser spatial resolution (*path 2*). Because of both the nonlinearity of  $f$  and the spatial heterogeneity of the coarser spatial resolution pixel, the difference between  $LAI^{exa}$  and  $LAI^{app}$  at each scale is defined as scaling bias. This study proposed a LAI scaling transfer model based on the fractal theory for computing LAIs at different scales.

#### 2.2.1. The average LAI of the image calculation

On the assumption that the size of a remote sensing image is  $N*N$  pixels ( $N$  is required to be a composite number) and  $n$  is the divisor of  $N$ ,  $n*n$  pixels were aggregated as a pixel at scale  $n$  to ensure that all pixels of the

image were aggregated when the average LAI of the image was computed. The number of aggregated pixels at scale  $n$  is  $(N/n)^2$ . The spatial resolution of the image reduced, which is the up scaling process. Take the CCD image with the size of 512\*512 pixels used in this study for example, the transfer scales  $n$  include 2, 4, 8, 16, 32, 64, 128, 256 and 512, and the corresponding coarser spatial resolutions are 60m, 120m, 240m, 480m, 960m, 1.92km, 3.84km, 7.68km and 15.36km respectively. This amounts to computing the NDVI of the  $j$ th pixel at scale  $n$  ( $NDVI_{n,j}^{app}$ ) as the average of the high spatial resolution NDVI values  $NDVI_i$  :

$$NDVI_{n,j}^{app} = \frac{1}{n^2} \sum_{i=1}^{n^2} NDVI_i \quad (2)$$

where  $n^2$  is the number of 30 m high resolution pixels within the aggregated pixel (scale  $n$ ).  $NDVI_i$  is the NDVI value of the  $i$ th pixel at the high resolution. The application of  $f$  to  $NDVI_{n,j}^{app}$  leads to an approximated LAI value of the  $j$ th pixel  $LAI_{n,j}^{app}$  :

$$LAI_{n,j}^{app} = f(NDVI_{n,j}^{app}) \quad (3)$$

The approximated value  $LAI_n^{app}$  of the whole image at scale  $n$  is computed by aggregating all  $LAI_{n,j}^{app}$  :

$$LAI_n^{app} = \frac{1}{\left(\frac{N}{n}\right)^2} \sum_{j=1}^{\left(\frac{N}{n}\right)^2} LAI_{n,j}^{app} \quad (4)$$

Similarly, The exact LAI value  $LAI_{n,j}^{exa}$  of the  $j$ th pixel at the coarser spatial resolution  $n*30$  m is computed by first applying  $f$  to  $n^2$   $NDVI_i$  at the high spatial resolution and then by aggregating the results of the corresponding  $LAI_i$  (Eq. (5)) at the coarser spatial resolution. The exact value  $LAI_n^{exa}$  of the whole image at the coarser spatial resolution is computed by aggregating all  $LAI_{n,j}^{exa}$  (Eq. (6)).

$$LAI_{n,j}^{exa} = \frac{1}{n^2} \sum_{i=1}^{n^2} f(NDVI_i) \quad (5)$$

$$LAI_n^{exa} = \frac{1}{\left(\frac{N}{n}\right)^2} \sum_{j=1}^{\left(\frac{N}{n}\right)^2} LAI_{n,j}^{exa} = \frac{1}{\left(\frac{N}{n}\right)^2} \sum_{j=1}^{\left(\frac{N}{n}\right)^2} \left[ \frac{1}{n^2} \sum_{i=1}^{n^2} f(NDVI_i) \right] = \frac{1}{\left(\frac{N}{n}\right)^2} \frac{1}{n^2} \sum_{j=1}^{\left(\frac{N}{n}\right)^2} \left[ \sum_{i=1}^{n^2} f(NDVI_i) \right] = \frac{1}{N^2} \sum_{i=1}^{N^2} f(NDVI_i) \quad (6)$$

As shown in Eq. (6),  $LAI_n^{exa}$  is independent of scale  $n$ , which means  $LAI_n^{exa}$  at any scale are the same. In addition, the number of aggregated pixels at the coarser spatial resolution  $1*30$  m is equal to the number of pixels at the high resolution, indicating that  $LAI_1^{app}$  is the same with  $LAI_1^{exa}$  (Eq. (7)).

$$LAI_1^{app} = \frac{1}{\left(\frac{N}{1}\right)^2} \sum_{j=1}^{\left(\frac{N}{1}\right)^2} LAI_{1,j}^{app} = \frac{1}{N^2} \sum_{j=1}^{N^2} f(NDVI_{1,j}^{app}) = \frac{1}{N^2} \sum_{j=1}^{N^2} f(NDVI_j) = LAI_1^{exa} \quad (7)$$

Eq. (6) and (7) combined show that  $LAI_n^{app}$  equals the  $LAI_n^{exa}$  at any scale. Therefore, the scaling bias between  $LAI_n^{exa}$  and  $LAI_n^{app}$  at the scale  $n$  could be translated into the difference between  $LAI_1^{app}$  and  $LAI_n^{app}$  at the scale  $n$ , which could be obtained by quantitatively describing the transformation law of  $LAI_n^{app}$  with the scale  $n$  changing.

The  $LAI_n^{ratio}$  proposed in Zhang et al (2010) was computed for analyzing and comparing the accuracy of the LAI scaling transfer model established in this paper:

$$LAI_n^{ratio} = \sum_{j=1}^{\binom{N}{n}} \frac{LAI_{n,j}^{app}}{LAI_{n,j}^{exa}} \quad (8)$$

### 2.2.2. Scaling transfer model

The remote sensing images with different spatial resolutions possess the scale invariance and self-similarity characteristic (Penland et al., 1996; Zhang et al., 2010). Scale invariance and self-similarity are the basis of Fractal geometry found by Mandelbrot (Mandelbrot, 1975), and hence fractal theory could be used to study the scale bias of quantitative remote sensing products.

According to the definition of fractal dimension, the measurements and scales are power correlated when different scales are applied to measure the study object with fractal characteristic, which means  $LAI_n^{app}$  have a power-law dependence at the scale  $n$  in this study (Eq. (9)). The scaling transfer model is established by applying a log transformation on both sides of Eq. (9) (Eq. (10)). The fractal dimension  $D$  of the image is approximated by Eq. (11):

$$LAI_n^{app} = K * n^d \quad (9)$$

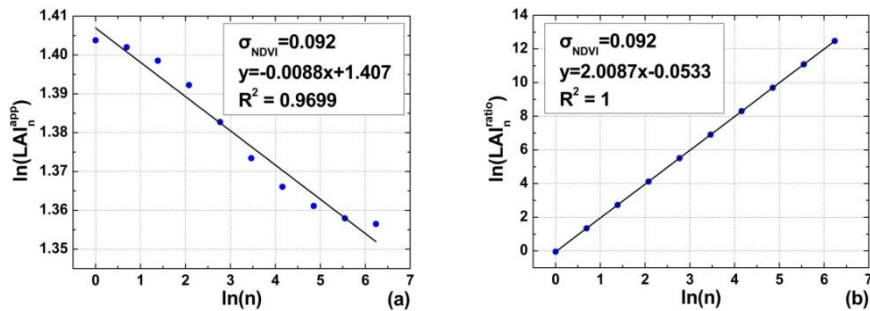
$$\ln(LAI_n^{app}) = d \ln(n) + \ln(K) \quad (10)$$

$$D = 2 - d \quad (11)$$

## 3. RESULTS

### 3.1. Scaling transfer model

As shown in Fig. 2a,  $LAI_n^{app}$  at each scale and scale  $n$  are double logarithmic linear correlated with  $R^2$  over 96%, which further indicated that the quantitative remote sensing products possess the fractal characteristic. The dimension fractal of the image ( $D=2.0088$ ) computed by the scaling transfer model is basically consistent with that ( $D=2.0087$ ) computed by the method proposed in Zhang et al (2010) (Fig. 2b). The  $LAI_n^{app}$  calculated with the scaling transfer model agree well with the ones calculated with the method of *path 2* with an APE value of 0.27% and a RMSE value of 0.0129 respectively (Fig. 2c). It could be concluded that the LAI scaling transfer model proposed in this study is feasible.



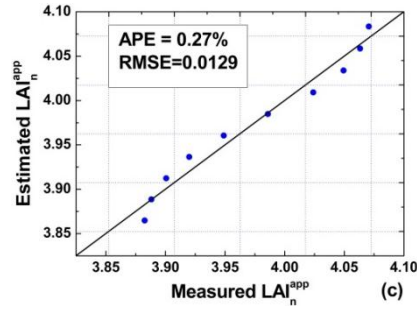


Figure 2 The fractal characteristic of inversion LAIs at different scales.

### 3.2. The influence of the standard deviation of the image on the scaling transfer model

In this study, the CCD image of size 512\*512 were divided into 1024 sub-images of size 16\*16, 256 sub-images of size 32\*32, 64 sub-images of 64\*64, 16 sub-images of 128\*128 and 4 sub-images of 256\*256, respectively. The relationships between the standard deviation  $\sigma_{NDVI}$  and fractal dimension  $D$  of each sub-image were given in Fig. 3. Regardless of the size of the sub-image, the fractal dimension of the sub-images increased as the  $\sigma_{NDVI}$  increased. It indicated that the fractal dimension in the scaling transfer model was mainly influenced by  $\sigma_{NDVI}$ .

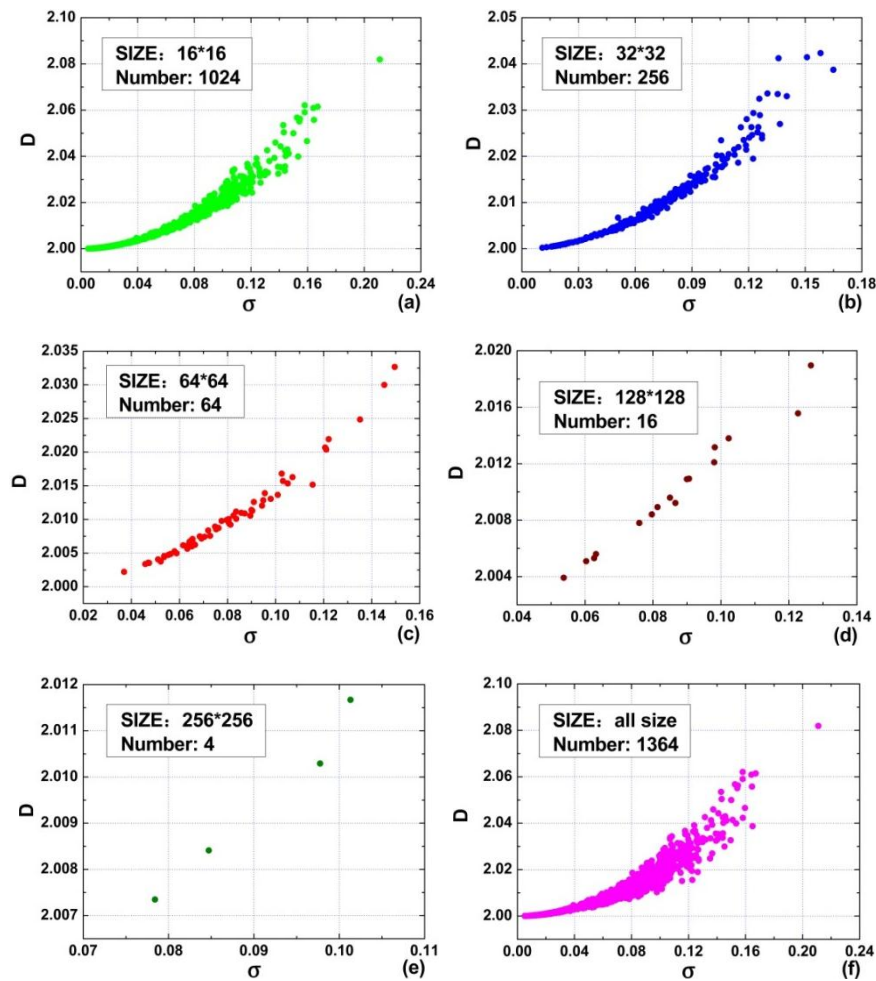


Figure 3 The relationship between the standard deviation  $\sigma_{NDVI}$  and the fractal dimension  $D$  of the sub-images with different sizes.

#### 4. DISCUSSION AND CONCLUSION

In this study, a LAI scaling transfer model based on fractal theory for computing LAIs at different scales was proposed. The results showed that the scaling transfer model performed well in estimating LAIs at different scales. The study found that the fractal dimension  $D$  of the image, the parameter of the scaling transfer model, increased as the standard deviation  $\sigma NDVI$  increased.

An image of size  $N*N$  was selected to perform the scaling transformation in this paper, and  $N$  is required to be a composite number. However, in the practical applications, the object of scaling transformation is mainly a certain land cover type, the shape of which is generally irregular. Therefore, further analysis and exploration are needed for the application of this method on the irregular objects in practice.

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