APPROXIMATION EQUATION OF EROSION CALCULATION IN GIS

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ABSTRACT: Surface erosion at a watershed scale is often estimated using the Universal Soil Loss Equation (USLE). During the erosion study of Shihmen reservoir watershed in Taiwan, we discovered a longstanding order-of-magnitude discrepancy in the literature**.** In order to explain the differences in the calculated amounts of erosion, we identified the following equation as one of the possible reasons:

$$\frac{\sum_{i=1}^{n} R_i k_i L_i S_i C_i P_i}{n} \approx \frac{\sum_{i=1}^{n} R_i}{n}\frac{\sum_{i=1}^{n} K_i}{n}\frac{\sum_{i=1}^{n} L_i}{n}\frac{\sum_{i=1}^{n} S_i}{n}\frac{\sum_{i=1}^{n} C_i}{n}\frac{\sum_{i=1}^{n} P_i}{n}$$

where *R*, *K*, *L*, *S*, *C*, and *P* are factors of USLE and *n* is the total number of cells used in GIS. In this study, we showed the derivation of the equation and used numerical simulations to test if the equation held true for Shihmen reservoir watershed.

1. INTRODUCTION AND MOTIVATION

Using GIS and USLE (Universal Soil Loss Equation) to evaluate the surface soil erosion of Shihmen reservoir watershed has been done before. However, researchers did not agree on the amount of erosion. The estimated amount of soil erosion ranged from about 1 ton/ha/year (Chiang et al., 2007) to 3310 ton/ha/year (Chen et al., 2009). There were a number of elements such as the assumed unit weight of the soil/sediment and the area of the watershed that could affect the results. In this study, we did not attempt to resolve this longstanding order-of-magnitude discrepancy in the literature. Instead, we aimed to focus on one aspect of the GIS computation that seemed to be confusing and counter-intuitive

2. PAST RESULTS

Our study was based on the Universal Soil Loss Equation (USLE), which is defined as follows (Gray and Sotir, 1996; Wu et al., 1996):

$$A_m = R_m \times K_m \times L \times S \times C \times P \tag{1}$$

where

- A_m: computed soil loss per unit area for a given time interval (tonne/hectare/year)
- R_m: rainfall factor (10^6joule-mm/hectare/hour/year)
- K_m: soil erodibility factor (tonne-hour/10^6joule/mm)
- L: slope length factor
- S: steepness factor
- C: vegetation factor
- P: erosion control practice factor

GIS map layers of R_m, K_m, L, S, C, and P factors of Shihmen reservoir watershed were previously created by Jhan et al. (2013) and Jhan (2014). Afterwards, Yang (2016) expanded and improved the system. Among the improvements, two new Digital Elevation Models (DEM) were added to the system—ASTER GDEM 2011 and CGS 2013. The comparison of all four DEMs are listed in Table 1. These DEMs were used in the calculation of L and S factors. Along with the other four factors, the soil erosion of Shihmen reservoir watershed was calculated. The results are shown in Table 2. Also shown in Tables 2 are the average values of R_m, K_m, L, S, and C factors, denoted as R_{avg}, K_{avg}, L_{avg}, S_{avg}, and C_{avg}. Note that P factor was not included because it was assumed to be one for conservative estimation.

In Table 2, the last column shows that calculated amounts of soil erosion by GIS, whereas the second to the last column shows the product of the average of R_m, K_m, L, S, and C factors. The product is,

$$R_{avg} \times K_{avg} \times L_{avg} \times S_{avg} \times C_{avg} \tag{2}$$

Table 1 Comparison of different DEMs used in erosion calculation

Types of DEM	Source	Technology	Resolution
CGS 2013	Central Geological Survey, Ministry of Economic Affairs of Taiwan	Airborne LiDAR	10 m
ASTER 2009	NASA and the Ministry of Economy, Trade, and Industry (METI) of Japan	Terra satellite	30 m
ASTER 2011	NASA and the Ministry of Economy, Trade, and Industry (METI) of Japan	Terra satellite	30 m
ASO 1985	Aerial Survey Office, Forestry Bureau, Council of Agriculture of Taiwan	Aerial photography	40 m

Table 2 Statistical data of map layers of erosion factors from GIS

Types of DEM	R_{avg} [1]	K_{avg} [2]	L_{avg} [3]	S_{avg} [4]	C_{avg} [5]	Average erosion [1]*[2]*[3]*[4]*[5] (ton/ha/year)	Average erosion from GIS (ton/ha/year)
CGS 2013	12786.55	0.02	0.82	24.94	0.01	52.30	43.24 (-17.32%)
ASTER 2009	12786.10	0.02	1.35	16.50	0.01	56.96	73.16 (28.44%)
ASTER 2011	12786.10	0.02	1.36	17.96	0.01	62.46	69.06 (10.56%)
ASO 1985	12785.47	0.02	1.59	20.06	0.01	81.56	93.90 (15.13%)

3. RESEARCH METHOD

Our intuition was that the result of equation (2) should be very close, if not identical, to the computed amount of soil erosion from GIS (last column of Table 2). However, Table 2 shows the contrary. Not only are the values not close to each other, but they also differ by a big margin. In extreme cases, the difference is as low as -17.32% and as high as 28.44%. Among four DEMs, the biggest difference in absolute amount is found in ASTER GDEM 2009 (version 1). Therefore, we decided to explore the reason for such a difference using ASTER GDEM 2009 with two methods. First, we attempted to derive an equation showing the relationship between the last two columns of Table 2. Second, we wrote a C++ program with the known values of R_{avg}, K_{avg}, L_{avg}, S_{avg}, and C_{avg} to simulate the results that were calculated by ArcGIS.

Since ASTER GDEM 2009 (version 1) generated the largest discrepancy, it was reasonable to use it in the following analysis. First, we computed and plotted the erosion map of Shihmen reservoir watershed as shown in Figure 1. The erosion amount of Figure 1 was computed for each cell (or grid) from corresponding values of R_m, K_m, L, S, and C factors. Note that the area of a cell is:

$$30 \times 30 = 900\ m^2 \tag{3}$$

Therefore, we applied the following ratio to each cell:

$$R_m \times K_m \times L \times S \times C \times P \times \frac{30 \times 30}{100 \times 100} \tag{4}$$

For comparison with other research results of the same watershed, the units in Figure 1are not ideal. Therefore, we resampled Figure 1 and changed the cell size to 100 m. Figure 2 shows the results. The area of each cell in Figure 2 is:

$$100 \times 100 = 10000\ m^2 = 1\ ha \tag{5}$$

Comparing Figure 1 with Figure 2, it is obvious that the amounts of erosion seemed to have increased. However, note that their units were different (ton/cell/year vs. ton/ha/year). Direct comparison between these two figures should not be made.

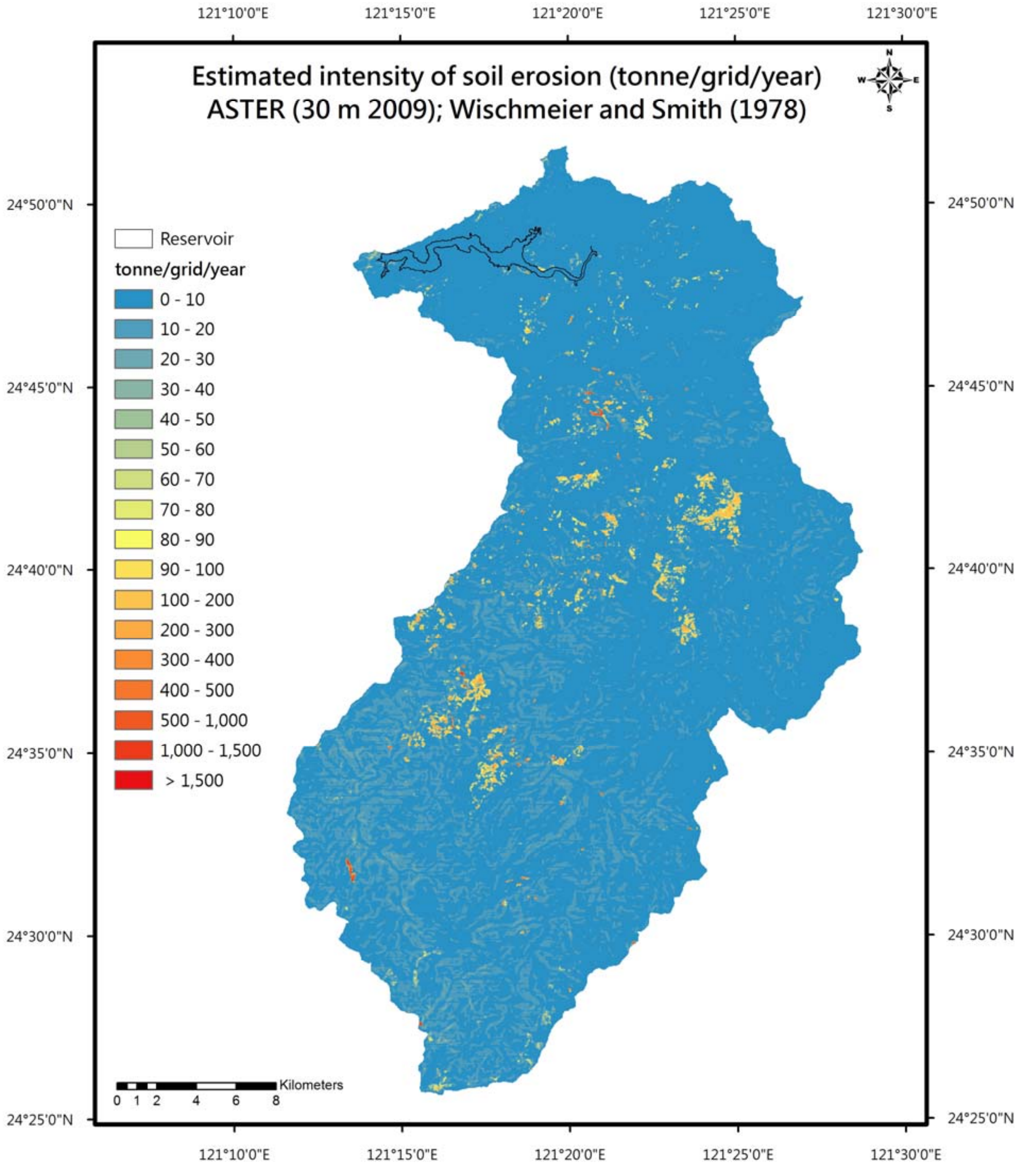

Figure 1 The map of erosion of Shihmen reservoir watershed (average erosion/cell = 6.58 ton/cell/year despite some extreme values on the map)

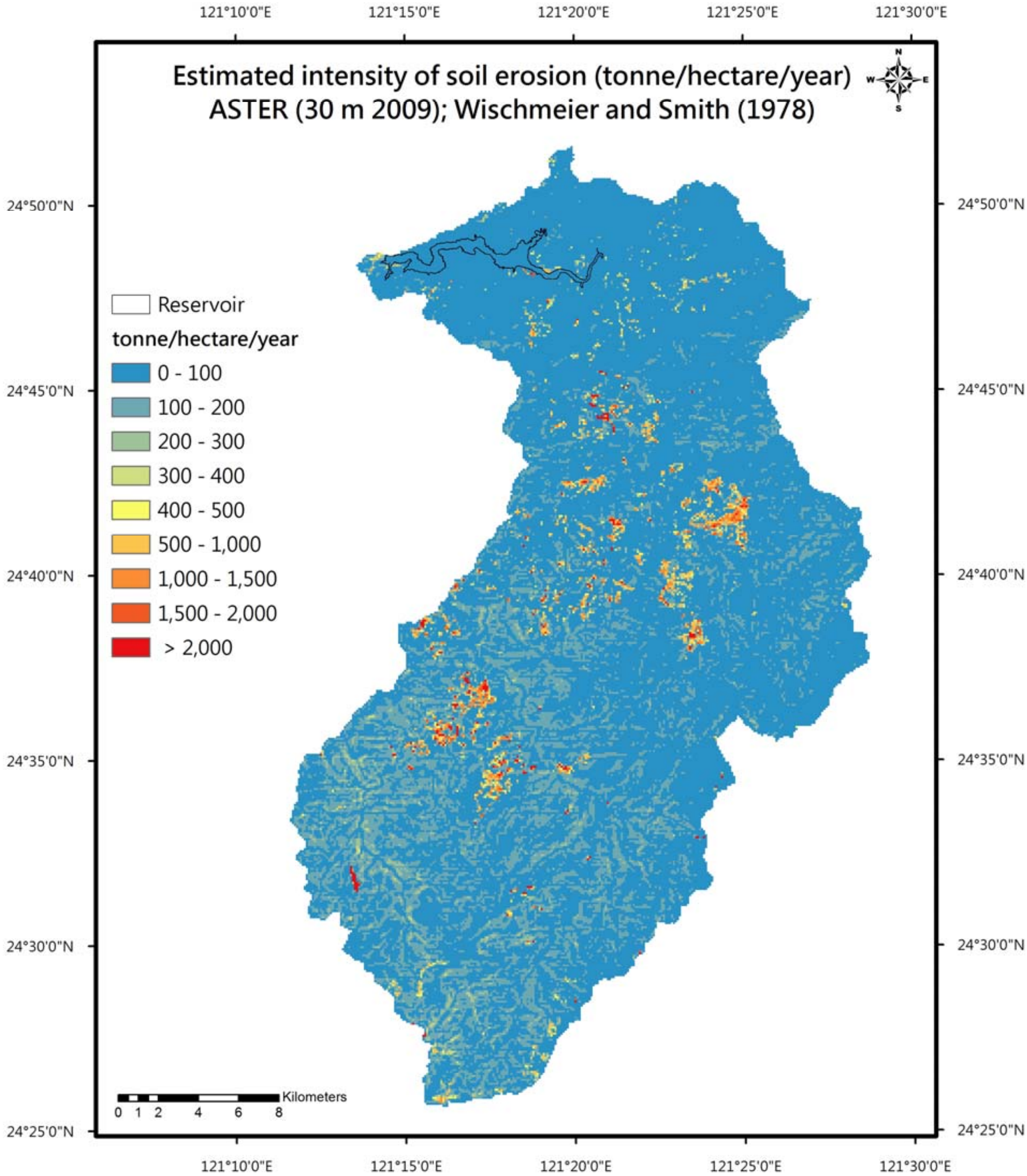

Figure 2 The map of erosion of Shihmen reservoir watershed (average erosion/ha = 73.16 ton/ha/year despite some extreme values on the map)

4. RESULTS

We explain the two approaches that we implemented in the following sections.

4.1 Equation Derivation

First, we examined the mathematical equations to see if they were of any help. Remember from USLE that A_m represents the total amount of erosion. The average erosion of an area could be calculated as follows:

$$A_{avg} = \frac{R_m \times K_m \times L \times S \times C \times P}{Area} \tag{6}$$

For a watershed that is divided into many cells, equations (6) was equal to:

$$A_{avg} = \frac{\sum_{i=1}^{n}\left((R_i k_i L_i S_i C_i P_i) \times \frac{A_{cell}}{100 \times 100m^2}\right)}{\frac{\sum A_{cell}}{100 \times 100m^2}} \tag{7}$$

$$= \frac{A_{cell} \times \sum_{i=1}^{n}(R_i k_i L_i S_i C_i P_i)}{n \times A_{cell}} \tag{8}$$

$$= \frac{\sum_{i=1}^{n} R_i k_i L_i S_i C_i P_i}{n} \tag{9}$$

where A_{cell} represents the area of each individual cell (same sizes):

$$A_{cell} = 30 \times 30 = 900\ m^2 \tag{10}$$

Since A_{cell} was a constant, it could be cancelled out from equation (8), and we arrived at equation (9). Here an added benefit was discovered. We did not have to consider the area of individual cells in order to compute the overall average erosion. We could simply take the average of every cell. The result was automatically expressed as ton/ha/year. The area was not given in km^2 or any other units because the R_m factor was given as 10^6joule-mm/hectare/hour/year. The USLE calculation automatically had hectares embedded in the equation.

Equation (9) now gives us the average erosion of the watershed. Note that in equation (9) the average erosion is the average of every cell. Is it possible to calculate the average of individual factors first and then multiply them together to obtain the average erosion? This way multiplications could be replaced by additions, and it should substantially speed up the computation. In other words, is the following equation true?

$$\frac{\sum_{i=1}^{n} R_i k_i L_i S_i C_i P_i}{n} \approx \frac{\sum_{i=1}^{n} R_i}{n} \frac{\sum_{i=1}^{n} K_i}{n} \frac{\sum_{i=1}^{n} L_i}{n} \frac{\sum_{i=1}^{n} S_i}{n} \frac{\sum_{i=1}^{n} C_i}{n} \frac{\sum_{i=1}^{n} P_i}{n} \tag{11}$$

4.2 Computer Simulation

Mathematically it does not seem that equation (11) always holds true. That motivated us to test the equation by numerical simulation. We wanted to see if both sides of equation (11) were the same, and if they were not the same, to see how close they were. We took the ASTER 2009 data from Table 2 and wrote a C++ program to test the equality of equation (11). In the C++ program, we set R_{avg} = 12786.10, K_{avg} = 0.02, L_{avg} = 1.35, S_{avg} = 16.50, and C_{avg} = 0.01. P was ignored because it was always one. We also knew that there were 843940 cells (30 m by 30 m) from GIS, so we set the program loop to run 843, 940 times. For each run, random numbers uniformly distributed in given ranges determined the factors R, K, L, S, and C. For the first experiment, we set the ranges to:

$$\mathrm{R} \in [0.5R_{avg}, 1.5R_{avg}] \tag{12}$$

$$\mathrm{K} \in [0.5K_{avg}, 1.5K_{avg}] \tag{13}$$

$$\mathrm{L} \in [0.5L_{avg}, 1.5L_{avg}] \tag{14}$$

$$\mathrm{S} \in [0.5S_{avg}, 1.5S_{avg}] \tag{15}$$

$$\mathrm{C} \in [0.5C_{avg}, 1.5C_{avg}] \tag{16}$$

The results are shown in the first column of Table 3. For the second experiment, we expanded the ranges to:

$$\mathrm{R} \in [0.1R_{avg}, 1.9R_{avg}] \tag{17}$$

$$\mathrm{K} \in [0.1K_{avg}, 1.9K_{avg}] \tag{18}$$

$$\mathrm{L} \in [0.1L_{avg}, 1.9L_{avg}] \tag{19}$$

$$S \in [0.1S_{avg}, 1.9S_{avg}] \quad (20)$$

$$C \in [0.1C_{avg}, 1.9C_{avg}] \quad (21)$$

The results of 30 simulations are shown in the second column of Table 3. It can be seen from the table that all 30 simulations generated very similar results. They were very close to the product of the average factors:

$$12786.10 \times 0.02 \times 1.35 \times 16.50 \times 0.01 = 56.96 \quad (22)$$

As the simulation ranges grew, so did the range of erosion. The first column of Table 3 has a standard deviation of 0.05 ton/ha/year, whereas the second column has 0.08 ton/ha/year. Nevertheless, the average was always very different from the 73.16 ton/ha/year computed by GIS.

Table 3 Results of 30 simulation of random numbers

Simulation	Average erosion (ton/ha/year) Random erosion factors from 0.5(average) to 1.5(average)	Average erosion (ton/ha/year) Random erosion factors from 0.1(average) to 1.9(average)
Simulation #1	56.86	57.09
Simulation #2	56.99	56.92
Simulation #3	56.94	57.01
Simulation #4	56.96	56.84
Simulation #5	56.93	56.87
Simulation #6	56.84	56.96
Simulation #7	56.96	56.82
Simulation #8	56.94	56.99
Simulation #9	56.99	57.05
Simulation #10	57.00	56.94
Simulation #11	56.96	57.02
Simulation #12	56.96	56.95
Simulation #13	57.02	57.07
Simulation #14	56.87	57.01
Simulation #15	56.94	56.96
Simulation #16	57.00	57.00
Simulation #17	56.92	56.95
Simulation #18	57.02	56.98
Simulation #19	57.00	57.08
Simulation #20	56.91	57.03
Simulation #21	56.99	56.88
Simulation #22	56.96	56.93
Simulation #23	56.97	57.09
Simulation #24	56.92	56.91
Simulation #25	56.90	56.85
Simulation #26	56.86	57.09
Simulation #27	56.99	56.92
Simulation #28	56.94	57.01
Simulation #29	56.96	56.84
Simulation #30	56.93	56.87
Min	56.84	56.82
Max	57.02	57.09
Average	56.95	56.97
Standard deviation	0.05	0.08

5. SUMMARY AND CONCLUSION

In this study, we tried to find a simpler way to compute average erosion, and perhaps to use it to validate the result of GIS. Two different approaches were taken for the investigation. First, we tried to prove mathematically that the product of averages was the same as the average of products. They were not. Then, we used numerical simulation with statistical data from a real watershed to test how close both sides of equation (11) were. The results showed that the product of averages was very close to the average of products. However, the simulated results were very different from the result of GIS (56.96 ton/ha/year vs. 73.16 ton/ha/year). A possible explanation is that the distribution of R, K, L, S, and C factors are not uniform within their respective ranges as assumed by our program. Further study in the future is needed.

In conclusion, this study demonstrated that we could not use the product of averages (R_{avg}, K_{avg}, L_{avg}, S_{avg}, and C_{avg}) to validate the average of products (R_i, K_i, L_i, S_i, and C_i) from GIS. The difference was substantial and could be as high as 28.44%. The distribution of individual erosion factors could be the key to explain this discrepancy.

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