Using Surface Splines for GIS- ready Imagery

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Abstract： High resolution data has become an important source of data, before they can be integrating into a GIS database, It requires processing for ortho-rectification to generate image map with high accuracy and low cost。
Using surface splines interpolation for rectification quite different with traditional grid method in photogrammetric, To introduce surface splines is important.
In fact the actual named is mechanical surface splines
because it must add mechanical conditions to form the formula.
The main advantages of the surface splines are that the coordinate of the known points are not located in a rectangular array and the function may be differentiated in find slopes.
Surface splines are a mathematical tool to interpolation a function of two variables.
It base upon small deflection equation of an infinite plate，
its originally developed for interpolation wing deflection of aircraft 1972
by Harderaed and Desmarais contributed.

An example map size beyond the custom. Only five control points for rectification (60cm×200cm)
if the number of control points increase also the map size increasing large and easy operating.
Other in civilian world the UAV gives low cost aerial imaging obvious using surface splines
In cartographic is a tool for supporting geospatial decisions

1. General Background

In the field of surveying the view points of common knowledge for interpolation could write the equation observation as

$$L=AX+BT+n \text{ (noise)}$$

Where A given q×p matrix expressing the effect of the parameters X on the observation L.

B represents linear operator and comprises the q linear functional, q-vector.

Otherwise surface splines gives the same math type (ref. formula 4) so in photogrammetric using surface splines naturally.

In formula (4)， $\sum_{i=1}^{N} F r_i^2 \ln r_i^2$ also get the

character of the reproducing kernel k(x, y) in Hilbert space which can ensure the receiving of minimal norm solution for interpolation in photogrammetric.

As the famous professor Karl Kraus said that mathematical model (4) has determined the trend function and the part treated with least squares interpolation together which solved the problem in an elegant way.

2. Mathematical analysis.

The formula derivation goes very strict: as following

As we know that a linear spline is the solution of an equation based upon a small deflection of an infinite beam.

The surface splines are the deformation of an infinite plate bending only.

The differential equation relating bending deflection and the load of the plate is

$$D \nabla 4 W=P \qquad (1)$$

D= flexural rigidity of the plate

W(x, y) = lateral deflection

P = lateral load

bending deflection and the load of the plate is

Deflections are specified at N independent points (xi, Yi) i =1…N Here needs to know the point loads Pi at these N points. When the load is determined, the deflection will be too

For example, if we want to know the symmetrical deflection of the origin point when the load is P,
use the equation (1)

Where

X= rcos θ

Y=rsin θ

And we get

W(r) = A+ Br2+(P/16πD)r2.lnr2 (2)

In the equation (2) A, B are undeterminedcoefficien while P is point load.

If continue the derivation will also add two conditions;

Deflection is symmetrical at the points with load.

The surface splines will be flat if far away from the applied loads.

The deflection of the whole splines is the summat equations (2)

W(x, y)=∑A+ Br2+(P/16πD)r2.lnr2 (3)

And after deduced and combined

we get

$$W(x,y)=A+Bx+Cy+\sum_{i=1}^{N} F_i r_i^2 \ln r_i^2 \qquad (4)$$

Here ri2 =(x-xi)2+(y-yi)2，Using the equation (4) need to solve N+3 unknown quantities.

Here N=5,

so add 3 equilibrium equations and then form equations, and get the final answer.

In this way which can correct the image point by point while avoid its translating and rotating, that is due to the function of the equilibrium equations.

Formula (5) are equilibrium equations.

$$\sum Fi=\sum X\,i\,F_i=\sum Yi\,Fi=0 \qquad (5)$$

3. Function of the software

This software can make image map 50cm×50cm from high resolution data based on five control points. The name is " five points method " It's easy to operate, anyone who can use PC works with it smoothly: there's no special skill for operation and the data to enter is little. It's easy to find an appropriate PC, for any ordinary PC can do the job.

The processing steps are as following

(1) Scan image into PC

(2) Enter the coordinate of the control points

(3) Identify the control points on the image (recognition of control points)

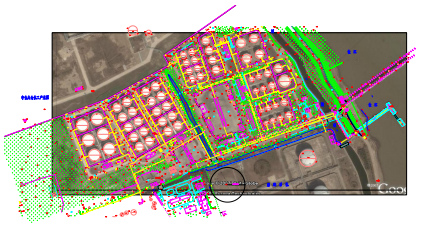
(4) Supply mapping scale

(5) Rectifications

4. Example

The map size is large beyond the custom size without any splices and modification.Using five control points for rectification from high resolution data to generate 1:500

Image map



5. Conclusion

Surface splines is useful in photogrammetric and survey it depends upon the solution of linear equations.

The closed form solution involves no function more than logarithms, and is easy coded.

Other using of interpolation was treating the residuals of block strip triangulation in photogrammetric also get successful.

Now, particular using surface splines in cartographic is a nice tool for supporting agriculture, forest, urban, geospatial decisions.

6. References

Robert L. Harder and Robert N. Desmaraist, Feb. 1972 Interpolation using surface spline, Aircraft Vol.9.No.2 Page 189-191

Wang Zhizhuo, July 1986 Continuation of concepts for Photogramme Page 184-187