Fuzzy Mixed Integer Programming for Land Use Planning in Tumauini, Isabela Philippines

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Abstract. Land use planning involves making decisions with multiple stakeholders that have conflicting and competing objectives, integration of assumptions and constraints that are ambiguous and inexact, and evaluation of land suitability. Land use plans for Tumauini, Isabela, Philippines were developed using fuzzy mixed integer goal programming models. All scenarios under the nonpreemptive structure resulted to the same land use plan. The proposed land use plan for nonpreemptive and preemptive structure required minimal land use reassignment. The proposed land use plan under preemptive priority structure achieved better suitability scores and had closer values to the target for every constraint.

1. INTRODUCTION AND LITERATURE REVIEW

Land is one of the many finite resources that sustains life on earth. Every parcel of land is useful to different stakeholders with competing and conflicting desires. For example, a parcel of land with verdant soil and is near residential areas can either be used as land for housing, for agriculture (small vegetable garden for the community) or for commercial establishments (community school, grocery store, etc.). Making decisions on use of land is more urgent nowadays since increasing population pressure and a mixed economy brings about greater competition between uses. United Nations Food and Agriculture Organization in 1976 (UNFAO) defined land use planning as the systematic assessment of land and water potential, alternative patterns of land use and other physical, social and economic conditions, for the purpose of selecting and adopting land use options which are most beneficial to land users without degrading the resources or the environment, together with the selection of measures most likely to encourage such land uses. In the Philippines, the government mandates every local government unit (LGU) to create a land use plan (LUP) adherent to the definition laid out by UNFAO. That is, a land use plan should meet the needs of today without compromising the ability of the future to meet its own. For example, an ideal land use plan strikes a balance for economic growth and residential area expansion due to the growing population without endangering protected areas and disrupting environmental conditions in the area.

Goal programming was used for plantation management in India [1, 2]. [3] used GP to determine compromise between gross margin maximization and expected risk for a LUP minimization. [4] is application of GP to forest planning. [5] used GIS and goal programming for land modelling in Indonesia while [6] used GP for regional agricultural planning. [7] integrated GIS and multicriteria analysis for agricultural land use in Netherlands.

A multi-choice mixed integer goal programming (MC-MIGP) model was formulated for the aggregate production planning of a Brazilian sugar and ethanol milling company [8]. The model in this study allowed decision makers to set multiple aspiration levels. [9] is a binary assignment problem with objectives of minimizing corridor land costs and amount of unsuitable land within corridor system.

2. Fuzzy Goal Programming

[10, 11] are few of the first applications of FGP. Most of the published applications of FGP are on forestry and agriculture. Forest managers deal with insufficient or imperfect information due to the innate complexity of forest systems. Also, some if not most of the market values of products are either not accurate or not available thus the reason for use of fuzzy concept. Thus the prevalent use of FGP as decision making tool in addressing the need to incorporate inaccuracy and ambiguity in available information [12, 13, 14, 15, 16, 17, 18, 19]. [20] did forest planning under fuzzy environment. Bare and Mendoza used FGP for timber harvest scheduling in 1992 [16] and for forest planning together with Z. Zhou on 1993 [17]. [21] used MOLP in resource management planning in 1994 and [22] for fuzzy environment in forest planning. [18] is an application of FMOLP for forest resource allocation. [23] is one of the fuzzy mixed integer goal programming applications for a machine scheduling problem.

Biswas and Pal used FGP to obtain a land use optimal production plan for several crops in District Nadia, West Bangal, India in 2005 where they created several scenarios following a preemptive structure of objectives and chose among the scenarios using Euclidean distance function [24]. In 2008, Mohaddes et. al. created a fuzzy environmental-economic model for land use planning. In this model, they considered economic, social and environmental objectives [25].

Barik (2015) created a linearly constrained probabilistic fuzzy goal programming model where the right hand side parameters in some constraints follow Pareto distribution with known mean and variance [26]. Aspiration levels are treated fuzzy and different additive approaches were used in aggregating the membership values that leads to deterministic models which can be solved using conventional LP solving techniques. One of the most notable uses of fuzzy set theory to agriculture is in [27, 28]. These authors thought that a fuzzy logic method is suitable for an accurate land suitability evaluation in FAO framework.

Other applications of multiobjective programming are as such. [29] is an application of fuzzy linear multiobjective programming in transportation. The model considers fuzzy right hand side coefficients and are expressed as triangular fuzzy numbers (TFN). It considers the objectives of minimizing total weighted flow time and total weighted tardiness. [30] used fuzzy goal programming to consider the imprecision of the target values and priorities of multiple objectives. [31] is an application of FGP to multiobjective transportation problems using linear and nonlinear (hyperbolic and exponential) membership functions. [26] presented a fuzzy goal programming formulation with right hand side parameters following Pareto distribution with known mean and variance, and fuzzy aspiration levels. [12] is another application of FGP for production planning with objectives of minimizing total costs in production, work force, inventory and rates of changes in work forces while [32] used it in watershed planning. [33] is an application of FGP for pollutant load calculation. [34] is an FMOP application for watershed management. [35] used FMIGP to determine optimal combination of design requirement values. [36] had optimized four fuzzy goals and several nonlinear constraints in an equipment purchasing problem using FMIGP.

Though FGP has been used to assess and craft development plans in specific sectors of the society, FGP seems to be sporadically used in the Philippines as there are no published literature on the use of FGP in land use planning in the country. On the other hand, several studies around the world have made progress in using various multiple criteria decision making (MCDM) techniques like FGP for land use planning.

[37] solved a fuzzy linear programming problem with linear membership functions using fuzzy decisive set method and modified sub-gradient method. In 2012, Gupta and Bhattacharjee [38] proposed two new methods in finding the solution to fuzzy goal programming problems and compared the results to the classic method introduced by Bellman and Zadeh in 1970 [39]. Shortly after, Kumar and Pal developed the method of incorporating fuzzy penalty functions to the fuzzy goal programming formulation to solve multiobjective problems [40]. [41] showed how fuzzy programming problems exhibiting dynamic programming characteristics can be solved by means of formulating the problem as a preemptive priority goal programming problem. [42] presented an additive FGP model that integrates different importance and preemptive priorities. The proposed approach exhibited better computational results compared to existing approaches. [43] proposed a novel fuzzy goal programming method where the hierarchy of goals may not be clearly defined.

This thesis in particular focused on land use planning for the municipality of Tumauini in the province of Isabela. The municipality has 46 barangays with a total land area (including bodies of water) of 46,730 hectares. Major ecosystems in the area are agriculture and forest. Though the main economic activity of Tumauini is agricultural, the municipality has also been recently experiencing a rapid increase in commercial activities.

3. CONCEPTUAL FRAMEWORK

3.1. Fuzzy Set Theory

Let U be a universal set and a set $A \subseteq U$. Unlike an ordinary mathematical set, a fuzzy set is defined by a membership function, often denoted by μ, that takes on function values ranging within the interval $[0, 1]$. If A is considered a fuzzy set, then the degree of membership of x to A, denoted by $\mu_A(x)$ can be any value between 0 and 1 inclusive. A higher degree of membership of x in A implies a closer value of $\mu_A(x)$ to 1. With the definition of a membership function, fuzzy sets allow partial membership of elements in a (fuzzy) set.

3.2. Fuzzy Set Theory in Goal Programming

Fuzzy set theory was integrated in goal programming formulations by using membership functions that allow inexactness, ambiguity and imprecision of parameters in the model.

The conventional goal programming approach converts a multiple (linear) objective problem to a conventional (linear) single objective problem by assigning goals to each of the objectives and combining all these goals in a single objective function [44]. Both objectives and constraints can be treated as fuzzy and will take the form of one of the fuzzy goal types.

3.3. Fuzzy Goal Types

Fuzzy goals can either be "$\lesssim$" (at most), "$\gtrsim$" (at least) or "$\approx$" (almost equal to). In the discussion, we will only consider the first two types: $AX \lesssim b$ or $AX \gtrsim b$.

If t_l is the allowed tolerance for the l^{th} fuzzy goal of the form $AX_l \lesssim b_l$, then the membership function is

$$\mu(AX_l) = \begin{cases} 1 & \text{if} \quad AX_l \leq b_l \\ 1 - \dfrac{AX_l - b_l}{t_l} & \text{if} \quad B \leq AX_l \leq b_l + t_l \\ 0 & \text{if} \quad AX_l > b_l + t_l \end{cases}$$

Similarly, if t_g is the tolerance for g^{th} fuzzy goal of the form $AX_g \gtrsim b_g$, the membership function is

$$\mu(AX_g) = \begin{cases} 1 & \text{if} \quad AX_g \geq b_g \\ 1 - \dfrac{b_g - AX_g}{t_g} & \text{if} \quad b_g - t_g \leq AX_g \leq b_g \\ 0 & \text{if} \quad AX_g < b_g - t_g \end{cases}$$

The general fuzzy goal programming formulation can be defined as

Maximize

$$\sum_{l=1}^{L} \mu((AX)_l) + \sum_{g=1}^{G} \mu((AX)_g) \tag{1}$$

subject to

$$\begin{aligned} \mu((AX)_l) &\leq 1 - \frac{(AX)_l - b_l}{t_l}, \forall l \in \{1, ..., L\} \\ \mu((AX)_g) &\geq 1 - \frac{b_g - AX}{t_g}, \forall g \in \{1, ..., G\} \\ X \geq 0, \mu((AX)_l) &\geq 0, \mu((AX)_g) \geq 0 \end{aligned}$$

The objective function can be modified such that weights are assigned to each of the fuzzy goals. Then Equation 1 becomes

$$\sum_{l=1}^{L} \omega_l \mu((AX)_l) + \sum_{g=1}^{G} \omega_g \mu((AX)_g) \tag{2}$$

where ω_l and ω_g is the weight assigned to l^{th} and g^{th} fuzzy goal, respectively.

4. MODEL FORMULATION

What makes every general model powerful is its flexibility to be modified and tailor-fitted according to the assumptions and data of a particular problem. As many objectives and constraints can be integrated in the model provided the availability of data. More importantly, FGP model can accommodate assumptions on imprecision and uncertainty.

The general modelling process using fuzzy goal programming can be summarized in these steps:

1. Define problem and enumerate data needed
2. Define each objective and constraint
3. Assign target value(goal) and tolerance to each fuzzy objectives and constraint
4. Formulate objective function

4.1. Data used in the model

Data were gathered from the municipality officers of Tumauini. They provided maps of every barangay classified according to land use and maps of different suitability criteria used in the model. They were also interviewed to determine weights for each objective and constraint.

4.2. Identify and define objectives and constraints

4.2.1. Identification and definition of variables The variables in the general model are defined as follows

B	–	set of barangays, $B = \{brgy_b \| i = 1, 2, ..., 46\}$
I_b	–	set of partitions with respect to barangay b, $I_b = \{i \| i = 1, 2, ..., P_b\}$, P_b is number of partitions in barangay b
J	–	set of land uses, $J = \{landuse_j \| j = 1, 2, ..., 6\}$
G	–	set of goals, $G = \{G_g \| g = 1, 2, 3, 4\}$
S_{gbij}	–	suitability score with g^{th} goal of partition i in barangay b given land use j
D_{bij}	–	assignment value $\{0, 1\}$ of partition i in barangay b given land use j
$area_{bi}$	–	area of partition i in barangay b (hectare)
tax_{bj}	–	amount per hectare for basic tax from barangay b given land use j (PHP)
PD	–	population density for every hectare of residential land (people)
SR	–	minimum number of officially registered residents in Tumauini (people)
AH	–	average harvest from a hectare of rice or corn (metric ton)
APR	–	minimum annual combined harvest from rice and corn (metric ton)
APR	–	minimum target for agricultural production requirement constraint (hectare)
BTR	–	minimum target for basic tax requirement constraint (hectare)
AA	–	land area (excluding water bodies) available in Tumauini (hectare)
$AVSS_1$	–	minimum average of suitability ratings of objective 1
$AVSS_2$	–	minimum average of suitability ratings of objective 2
$AVSS_3$	–	minimum average of suitability ratings of objective 3
$AVSS_4$	–	minimum average of suitability ratings of objective 4

4.2.2. Fuzzy objectives description and formulation These were the objectives considered in the model. All objectives are considered fuzzy.

1. Maximize disaster aversion - Priority on disaster aversion for residents and the built up areas is prioritized in this goal.

$$\sum_{b=1}^{46} \sum_{i=1}^{P_b} \sum_{j=1}^{6} S_{1bij} D_{bij} \tag{3}$$

2. Maximize agricultural production - Isabela is the second largest producer of agricultural crops. Rice and corn production should be maximized.

$$\sum_{b=1}^{46} \sum_{i=1}^{P_b} \sum_{j=1}^{6} S_{2bij} D_{bij} \tag{4}$$

3. Maximize economic activity - Tumauini has started to gain more economic activities. Land assigned for economic activities should be prioritized to maximize the economic gain of Tumauini.

$$\sum_{b=1}^{46} \sum_{i=1}^{P_b} \sum_{j=1}^{6} S_{3bij} D_{bij} \tag{5}$$

4. Maximize environmental protection - portion of Sierra Madre Natural Park is under Tumauini's jurisdiction. Environmental protection is a priority.

$$\sum_{b=1}^{46} \sum_{i=1}^{P_b} \sum_{j=1}^{6} S_{4bij} D_{bij} \tag{6}$$

4.2.3. Fuzzy constraints description and formulation All these constraints have fuzzy right hand side values.

1. Population requirement
   The mayor of Tumauini aims to achieve a population of at least 100,000 by 2025. By considering the current population density per residential hectare, the constraint is formulated as such

$$\sum_{b=1}^{46}\sum_{i=1}^{P_b} PD \cdot area_{bi} \cdot D_{bij} \gtrsim SR \tag{7}$$

   where value of j corresponds to residential land use
2. Rice and corn production requirement
   Main agricultural crops produced in Tumauini are rice and corn. Although there are other high value crops in the area, only the two main crops are considered. Assessment of land suitability for agriculture was done regardless of the crop to be planted in the area. Crop rotation is allowed in any assigned agricultural land.

$$\sum_{b=1}^{46}\sum_{i=1}^{P_b} AH \cdot area_{bi} \cdot D_{bij} \gtrsim APR \tag{8}$$

   where value of j corresponds to agricultural land use
3. Basic tax collection requirement
   With the mayor's aim of having Tumauini accredited as a city in 10 years, the municipality has to have at least PHP 100,000,000 annual income. Suppose 10% of this will come from basic tax, the constraint is formulated as

$$\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} tax_{bj} \cdot area_{bi} \cdot D_{bij} \gtrsim BTR \tag{9}$$

4. Total area available
   The total of land portions assigned to a specific land use should not exceed the total area available in Tumauini. This does not include water bodies.

$$\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} area_{bi} \cdot D_{bij} \leq AA \tag{10}$$

5. Land portion assignment constraints
   Every portion of land should only have one land use.

$$\sum_{j=1}^{6} D_{bij} = 1 \;\forall b \in \{1, 2, ..., 46\}, i \in \{1, ..., P_b\} \tag{11}$$

4.3. Formulation of general fuzzy mixed integer goal programming model

4.3.1. Transformation of goals to fuzzy goals Goals will be transformed to fuzzy goals depending on whether they were maximizing or minimizing goals. In this problem, all were maximization objectives. Maximizing goals, say MAXZ, were transformed to $Z \gtrsim T$ where $\gtrsim$

means " essentially greater than or equal to". This means that a greater objective function value is adequate but a lower value is only acceptable if it does not exceed the tolerance value.

The objectives in the study were transformed such that all the goals aim to maximize their average suitability score. The fuzzy goals were the following:

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{1bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} \gtrsim AVSS_1 \qquad \text{Disaster aversion}$$

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{2bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} \gtrsim AVSS_2 \qquad \text{Agricultural production}$$

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{3bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} \gtrsim AVSS_3 \qquad \text{Economic activity}$$

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{4bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} \gtrsim AVSS_4 \qquad \text{Environmental protection}$$

4.3.2. Transformation of fuzzy goals to linear inequalities One of the benefits in fuzzy goal programming is its ability to incorporate tolerance in the model. This tolerance is the allowable deviation from the target value. This will be provided by the decision-makers thus making it more adaptive to their interests. Since all the fuzzy goals are of type $z_g(x) \gtrsim t_g$, let lg_g be the lower tolerance limit of g^{th} goal. Its membership function is defined as

$$\mu(Z_g(x)) = \begin{cases} 1 & \text{if} \quad t_g \leq z_g(x) \\ 1 - \dfrac{t_g - z_g(x)}{lg_g} & \text{if} \quad t_g - lg_g \leq z_g(x) < t_g \\ 0 & \text{if} \quad z_g(x) < t_g - lg_g \end{cases}$$

This shows that the membership grade is between 0 and 1 inclusive.

Based on the membership function for this type of goal, let

$$\lambda_{G_g} \leq 1 - \frac{t_g - z_g(x)}{lg_g} \Rightarrow t_g \leq lg_g\left(1 - \lambda_{G_g}\right) + z_g(x)$$

The fuzzy goals are now defined as follows:

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{1bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} + lg_1\left(1-\lambda_{G_1}\right) \geq AVSS_1 \tag{12}$$

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{2bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} + lg_2\left(1-\lambda_{G_2}\right) \geq AVSS_2 \tag{13}$$

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{3bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} + lg_3\left(1-\lambda_{G_3}\right) \geq AVSS_3 \tag{14}$$

$$\frac{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} S_{4bij}D_{bij}}{\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} D_{bij}} + lg_4\left(1-\lambda_{G_4}\right) \geq AVSS_4 \tag{15}$$

4.3.3. Transformation of fuzzy constraints to linear inequalities Since constraints usually have fixed right-hand side values, tolerance levels were added to allow deviation within a specific range from these fixed values just like with the fuzzy goals. Since all fuzzy constraints are of the form $AX_c \gtrsim B_c$, we focus only on the formulation of this type.

A membership function for fuzzy constraints of the form $AX_c \gtrsim B_c$ where lc_c is the allowed deviation from B_c was defined as

$$\mu(AX) = \begin{cases} 1 & \text{if} \quad AX_c \geq B_c \\ 1 - \dfrac{B_c - AX_c}{lc_c} & \text{if} \quad B_c - lc_c \leq AX_c < B_c \\ 0 & \text{if} \quad AX_c < B_c - lc_c \end{cases}$$

Let

$$\lambda_{C_c} \leq 1 - \frac{B_c - AX_c}{lc_c} \Rightarrow B_c \leq lc_c\left(1-\lambda_{C_c}\right) + AX_c$$

Hence, the fuzzy constraints were transformed as follows:

$$\sum_{b=1}^{46}\sum_{i=1}^{P_b} PD \cdot area_{bi} \cdot D_{bij} + lc_1\left(1-\lambda_{C_1}\right) \geq SR \tag{16}$$

$$\sum_{b=1}^{46}\sum_{i=1}^{P_b} AH \cdot area_{bi} \cdot D_{bij} + lc_2\left(1-\lambda_{C_2}\right) \geq APR \tag{17}$$

$$\sum_{b=1}^{46}\sum_{i=1}^{P_b}\sum_{j=1}^{6} tax_{bj} \cdot area_{bi} \cdot D_{bij} + lc_3\left(1-\lambda_{C_3}\right) \geq BTR \tag{18}$$

4.4. Formulation of fuzzy mixed integer goal programming model under nonpreemptive priority structure

The objective of fuzzy goal programming is to find a solution that maximizes membership of all goals and constraints.

The weights represent the contribution of the goals and constraints to the value of the objective function. In decision making, unless all weights are equal, the goals or constraints with greater weight are those that are more important to the decision maker. The mathematical model for this study were tested using equal and unequal weights as prescribed by the stakeholders.

The FMIGP model under nonpreemptive priority structure is given by

$$\begin{gathered} \text{MAXIMIZE } \sum_{g=1}^{4} \left(\omega_{\lambda_{G_g}} \lambda_{G_g} \right) + \sum_{c=1}^{3} \omega_{\lambda_{C_c}} \lambda_{C_c} \\ \text{subject to} \\ \text{Equations 12 to 15} \\ \text{Equations 16 to 18} \\ \text{Equation 10} \\ \text{Equation 11} \\ D_{bij}, \lambda_{G_i}, \lambda_{C_i} \geq 0 \ \forall b \in \{1, 2, ..., 46\}, i \in \{1, 2, ..., P_b\}, j \in \{1, 2, ..., 6\} \end{gathered}$$

4.5. Formulation of fuzzy mixed integer goal programming model under preemptive priority structure

The formulation for the fuzzy mixed integer goal programming model under preemptive priority structure in this thesis was based on the conventional GP formulation (Section **??** with the objective function slightly modified to accomodate flexibility in constructing priority levels. The conventional preemptive priority structure optimizes one objective at a time.

The objective function is redefined such that it maximizes membership in every priority level Pr_p where p is the number of priority levels. Since the priority structure follows the conventional preemptive priority structure, suppose that the priority structure is given by $G_a \succ G_b \succ ... \succ G_p$, Pr_p is defined as

$$\begin{aligned} Pr_1 &= \left(\omega_{\lambda_{G_a}} \lambda_{G_a} \right) + \sum_{\forall c} \left(\omega_{\lambda_{C_c}} \lambda_{C_c} \right) \\ Pr_2 &= \left(\omega_{\lambda_{G_b}} \lambda_{G_b} \right) \\ & ... \\ Pr_p &= \left(\omega_{\lambda_{G_p}} \lambda_{G_p} \right) \end{aligned}$$

Based on the priority formulation, the top priority level always includes all constraints together with one goal. The results (if any) in this level is added to the model for the next iteration. Succeeding priority levels, involve maximization of membership of a single goal. Hence, the final FMIGP model under preemptive priority structure is

$$\begin{gathered} \text{MAXIMIZE } Z = [Pr_1, Pr_2, ..., Pr_p] \\ \text{subject to} \\ \text{Equations 12 to 15} \\ \text{Equations 16 to 18} \\ \text{Equation 10} \\ \text{Equation 11} \\ D_{bij}, \lambda_{G_i}, \lambda_{C_i} \geq 0 \ \forall b \in \{1, 2, ..., 46\}, i \in \{1, 2, ..., P_b\}, j \in \{1, 2, ..., 6\} \end{gathered}$$

4.6. Determination of weights/priority structure of objectives

Fuzzy LinPreRa method was used to ensure the consistency of the judgment matrices. The fuzzy linguistic preference scale used for LinPreRa was based on the triangular fuzzy numbers.

These triangular fuzzy numbers correspond to Saaty's crisp 1-9 scale (Table **??**).

Table 1: Fuzzy numbers and its corresponding triangular fuzzy number and fuzzy linguistic preference scale

FUZZY NUMBER	TRIANGULAR			FUZZY LINGUISTIC PREFERENCE		
a_{ij}	l_{ij}	m_{ij}	r_{ij}	l_{ij}	m_{ij}	r_{ij}
9^{-1}	0.111	0.111	0.125	0.0	0.0	0.0
8^{-1}	0.111	0.125	0.143	0.0	0.0	0.1
7^{-1}	0.125	0.143	0.167	0.0	0.1	0.1
6^{-1}	0.143	0.167	0.2	0.1	0.1	0.1
5^{-1}	0.167	0.2	0.25	0.1	0.1	0.2
4^{-1}	0.2	0.25	0.333	0.1	0.2	0.3
3^{-1}	0.25	0.33	0.5	0.2	0.3	0.3
2^{-1}	0.33	0.5	1	0.3	0.3	0.5
1^{-1}	0.5	1	1	0.3	0.5	0.5
just equal	1	1	1	0.5	0.5	0.5
1	1	1	2	0.5	0.5	0.7
2	1	2	3	0.5	0.7	0.8
3	2	3	4	0.7	0.8	0.8
4	3	4	5	0.8	0.8	0.9
5	4	5	6	0.8	0.9	0.9
6	5	6	7	0.9	0.9	0.9
7	6	7	8	0.9	0.9	1.0
8	7	8	9	0.9	1.0	1.0
9	8	9	9	1.0	1.0	1.0

The original judgment matrices used the scale in Table **??** and transformed judgment matrices using Fuzzy LinPreRa Method.

The original and transformed matrices are found in the Appendix.

Table 2: Aggregated matrix

OBJECTIVE	DA			AP			EA			EP		
DA	0.50	0.50	0.50	0.14	0.41	0.75	0.43	0.70	1.00	0.43	0.57	0.85
AP	0.25	0.53	0.86	0.50	0.50	0.50	0.69	0.77	0.86	0.43	0.61	1.00
EA	0.00	0.24	0.57	0.14	0.22	0.31	0.50	0.50	0.50	0.14	0.31	0.75
EP	0.15	0.41	0.57	0.00	0.00	0.57	0.25	0.60	0.86	0.50	0.50	0.50

Aggregation of matrices was done using method of Chang et. al. (2009) [45].

$V(M_2 \geq M_1)$

$$
\begin{array}{ll}
V(DA \geq AP) = 0.543189644 & V(EA \geq DA) = 0.699620547 \\
V(DA \geq EA) = 1 & V(EA \geq AP) = 0.620097846 \\
V(DA \geq EP) = 1 & V(EA \geq EP) = 0.914491688 \\
V(AP \geq DA) = 1 & V(EP \geq DA) = 0.797754055 \\
V(AP \geq EA) = 1 & V(EP \geq AP) = 0.725816674 \\
V(AP \geq EP) = 1 & V(EP \geq EA) = 1
\end{array}
$$

Determination of weights was done using FAHP method of Chang (1996) [46]. Based on the comparisons shown above, the computed normalized final weights of objectives are presented in the table below.

Table 3: Normalized final weights

OBJECTIVE	DA	AP	EA	EP
Weight	0.19	0.35	0.21	0.25

Among the objectives, agricultural production is the top priority followed by environmental protection and then economic activity. Least important is disaster aversion.

5. Validation of Models

The results of validating the FMIGP models are presented in this section. Models were ran using GUSEK (GLPK Under Scite Extended Kit) Version 0.2.18 using a laptop with a 1.6 GHz processor. FMIGP models under nonpreemptive priority structure took less than 5 seconds to solve while the FMIGP models under preemptive priority structure took 50 to 60 minutes.

5.1. Scenarios under nonpreemptive priority structure

The sum of weights ω_{G_g} and ω_{C_c} for all g,c is always equal to 1. The scenarios for the nonpreemptive priority structure are differentiated based on the weights associated with G_g's and C_c's. The weights are summarized as follows:

Table 4: Summary of weights for the four scenarios under nonpreemptive priority structure

WEIGHT/SCENARIO	1	2	3	4
$\omega_{\lambda_{G_1}}$	0.175	0.1	0.133	0.076
$\omega_{\lambda_{G_2}}$	0.175	0.1	0.245	0.14
$\omega_{\lambda_{G_3}}$	0.175	0.1	0.147	0.084
$\omega_{\lambda_{G_4}}$	0.175	0.1	0.175	0.1
$\omega_{\lambda_{C_1}}$	0.1	0.2	0.1	0.2
$\omega_{\lambda_{C_2}}$	0.1	0.2	0.1	0.2
$\omega_{\lambda_{C_3}}$	0.1	0.2	0.1	0.2

The weights are distributed in such a way that the total weights for goals and constraints are 0.7 and 0.3, respectively, for the first and third scenarios while it is 0.4 and 0.6, respectively, for the second and fourth scenarios. The goals and constraints in the first two scenarios share the total weight equally whereas the goals in the third and fourth scenarios are distributed according to the weights assigned to each goal as shown in Table 3.

5.2. Scenario under preemptive priority structure

For the preemptive priority structure, only the priority structure that follows the results in Section 4.6 was considered.

Table 5: Priority structure and weights of fuzzy objectives and constraints

GOAL/CONSTRAINT	PRIORITY LEVEL	WEIGHT AT PRIORITY LEVEL
Disaster Aversion	4	1
Agricultural Production	1	0.25
Economic Activity	3	1
Environmental Protection	2	1
Population Requirement	1	0.25
Rice and Corn Production	1	0.25
Basic Tax Collection	1	0.25

The table shows that the priority structure follows the conventional preemptive priority structure where one objective is optimized at a time and the constraints are part of the first priority level. Among the goals, agricultural production is the priority followed by environmental protection and then economic activity. Least prioritized is disaster aversion. Total of weights in each priority level is 1. In the first level, the weights are equally distributed among the fuzzy objectives and constraints.

5.3. Other parameters of the model

Table 6: Target and bounds of goals and constraints

GOAL/CONSTRAINT	TARGET	LOWER BOUND	UPPER BOUND
Ave. Suit. DA	0.5	-0.04	1
Ave. Suit. AP	0.5	-0.04	1
Ave. Suit. EA	0.5	-0.04	1
Ave. Suit. EP	0.5	-0.04	1
Pop'n Req't	100000	99000	-
Agri Prod (MT)	68794.65	68106.704	-
Basic Tax (PHP)	10000000	9900000	-
Area Used (HA)	-	44795.353	45553.62

The population density per residential area is set to 59.01434 people/residential hectare while average yield for rice and corn is 5 MT per hectare.

5.4. Proposed land use plans

Table shows the current land use plan alongside the proposed under nonpreemptive and preemptive priority structure. The assignment of every land partition is found in the Appendix.

All of the four scenarios under nonpreemptive priority structure resulted to the same land use plan shown under 'Proposed (NPS)' while the only scenario under preemptive priority structure is presented under 'Proposed (PPS)'. Note that the current land use plan do not satisfy the population and basic tax requirements.

Table 7: Current and proposed land use plans

ITEM/LUP	TARGET	CURRENT	PROPOSED (NPS)	PROPOSED (PPS)
Obj function	-		1	1
Ave. Suit. DA	0	0.86	0.81	0.85
Ave. Suit. AP	0	0.92	0.87	0.91
Ave. Suit. EA	0	0.81	0.75	0.89
Ave. Suit. EP	0	0.93	0.88	0.92
Pop'n Req't	100000	70207	100166	100907
Agri Prod (MT)	68794.65	68794.65	92239.655	69476.325
Basic Tax (PHP)	10000000	4503830.47	11397466.65	10085567
Area Used (HA)	-	44795.353	44795.353	44795.353
Land for RE	-	1189.661	1697.321	1709.871
Land for CO	-	12.931	281.381	211.006
Land for BU	-	4.7	4.7	514.191
Land for AG	-	13758.931	18447.931	13895.265
Land for FO	-	24364.02	24364.02	24364.02
Land for OP	-	5465.11	0	4101

Both proposed land use plans satisfied the targets without making use of the allowed deviation. This implies that if any of the two proposed land use plans is followed, Tumauini can be home to 100000 people while maintaining the same population density per residential hectare, agricultural production can be satisfied and 10% of local income can come from basic tax collection.

Proposed (NPS) has lower average land suitability scores compared to the current. Proposed (PPS) on the other hand has very close average land suitability scores but its average suitability score for economic activity is significantly higher than the current. This makes the land use assignments in proposed (PPS) closely the same with the current land use plan but with a significant number of land conversions done for commercial activities. Most of the land partitions converted to commercial areas are those currently considered as open areas.

Areas assigned to residential and built-up areas in Proposed (NPS) are closer to the current than the Proposed (PPS). On the other hand, Proposed (NPS) has greater allocation for commercial activities than Proposed (PPS) but it has lower average suitability in economic activity. This only means that though more areas were designated for commercial in Proposed (NPS), these areas may not be as suitable for commercial activities as those assigned in the Proposed (PPS). Forest areas should remain as they are for all proposed land use plans as supported by Republic Act 7586 (National Integrated Protected Areas System Act of 1992). Agricultural areas were significantly increased in Proposed (NPS) but it required conversion of all open areas to agricultural areas. This may not be an ideal choice because the planning period is only for 10 years. Conversion of land in the future may become more difficult if all open areas are converted to 'used' land early on. In Proposed (PPS), open areas were also converted but only to a minimum so that targets are met.

Overall, Proposed (PPS) has better suitability scores and more reasonable land assignments compared to Proposed (NPS).

6. CONCLUSION AND DISCUSSION

Fuzzy goal programming models were formulated to determine a land use plan (LUP) for every scenario in Tumauini, Isabela, Philippines. Identified stakeholders were able to provide most of the important data in formulating and validating the model. Current land use plan does not satisfy the targets. Based on the models, two land use plans were proposed, one from the nonpreemptive and another from the preemptive priority structure. Between the two proposed LUPs, the proposed land use plan from FMIGP model under preemptive priority structure (Proposed (PPS)) has closer suitability scores to the current than the proposed LUP under nonpreemptive priority structure (Proposed (NPS)). Moreover, in the Proposed (PPS), land use assignments are closer to the current and not all open areas were converted, making space for future land use changes.

7. RECOMMENDATIONS

The models can be modified such that other scenarios and assumptions are considered such as timber production. Other objectives that aim to minimize can be incorporated in the model as well as other constraints like migration rate. Other suitability critieria can also be added and the model can be tested with a different set of weights. More importantly, the general FMIGP models can be extended for other cities, municipalities or other areas that require land use planning.

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