

# Utilizing chaos equations and fractal dimensions to differentiate various crop types

Re-Yang Lee<sup>1</sup>, Chia-Hui Hsu<sup>1</sup>, Yi-Shiang Shiu<sup>2</sup>, I-Chen Yang<sup>1</sup>,

<sup>1</sup>Department of Land Management, Feng-Chia University

No. 100, Wenhwa Rd., Seatwen, Taichung, 40724 Taiwan (R.O.C.)

e-mail:rylee@fcu.edu.tw ; snoopy80729@gmail.com

<sup>2</sup>Department of Urban Planning and Spatial Information, Feng Chia University

No. 100, Wenhwa Rd., Seatwen, Taichung, 40724 Taiwan (R.O.C.)

e-mail:ysshui@fcu.edu.tw

**KEY WORDS:** Chaos, fractal, hyperspectral

Chaos theory is a method used in both qualitative and quantitative analysis for exploring the behavior of dynamic systems that can only be predicted using overall and continuous data relationships instead of single data relationships.

A chaotic algorithm requires entry of a series of data. Obtaining continuous data from current commercial satellites or aerial images is difficult because of climate conditions and budget constraints. However, hyperspectral ground images captured by portable devices exhibit numerous wavebands and narrow spectral ranges. Spectral ranges that are similar form continuous data. Spectral information in hyperspectral images is richer than that in ordinary multispectral images, rendering them useful for detecting minor spectral differences. Such differences overcome the insufficiency in multispectral images. Portable spectroradiometers can be used to measure the on-site reflectance curves of various land features at various times, locations, and statuses. Because crops change continuously and rapidly according to temporospatial conditions (e.g., etiolation, abscission, disease, unevenly distributed spatial density, and differences in planting times), a considerable number of variables are added to crop spectra, which is the primary cause of difficulty in classifying crops. Chaotic algorithms may be suitable for solving the crop classification problem and effectively identify various crop categories.

The research sample investigated in the present study comprised garlic, scallion, sweet potato, and carrots, most of which are planted in Yunlin County, Taiwan and are easy to sample. The chaotic image features of spectral reflectance of the various crops were captured at the same time and compared to discern any differences. MATLAB was used to conduct the chaos simulation and to calculate the fractal dimensions.

## 1.INTRODUCTION

Chaos is a movement pattern of a nonlinear system (Guess & Sailor, 1993). Strunk (1995) reported that chaos theory explains phenomenon previously rejected in natural science studies. Nonlinear systems exhibit irregular movement patterns under certain circumstances. Specifically, the systems appear to be irregular, but actually exhibit a certain degree of regularity in the irregularity. This phenomenon is called chaos, representing regularity in the presence of irregularity.

Chaos is essential in constructing and predicting time-series models. Chaos theory must be adopted to investigate targets that comprise the following three elements: (a) evolution equations, (b) system density, and (c) sensitive dependence on initial conditions. Feigenbaum (1983) claimed that a system exhibiting chaotic behavior is never in the same state twice. Events develop or change along a single direction. Although chaotic time-series seem stochastic, they are generated by a certain type of system. The generated data belong to a short-term prediction. A slight difference in the input data may affect the overall prediction results, thereby diverging from the original series. For instance, the butterfly effect explains how a slight difference or change in the initial values can be magnified or expanded through iteration, thereby resulting in system unpredictability (Lorenz, 1963).

The theory has been applied in numerous domains such as natural systems, climate models, fluid movements, migration trends, communication systems, business management (Trygestad, 1997), ecology (Kauffman, 1991), and economics (Kelsey, 1988). However, no study has applied chaos theory to remote sensing.

In the past, scientists investigated the chaos phenomenon in electronic circuit systems as a precursor to investigating chaos in real systems. The chaos phenomenon was originally calculated using mathematical algorithms in computers to predict climate change. Circuit systems exhibit high fitness to corresponding mathematical models, thereby facilitating simulating various circuit systems and repeating various complex nonlinear phenomena. Thus, exploring the chaos phenomenon from the perspective of electronic circuit systems is appropriate. Chua's circuit has been extensively employed in practical operations. Because it is a typical nonlinear chaotic circuit with a simple structure that can be easily performed in engineering experiments, chaos phenomena can be generated within appropriate parameter ranges. Thus, Chua's circuit was selected in the present study as a foundation for remote-sensing applications.

The strong winds and torrential rain from typhoons and extreme climates have had a considerably negative effect on agriculture in Taiwan. Natural disasters frequently damage crops and disrupt the balance of production and sales. Rising crop prices have a marked influence people's livelihoods. In addition, because farmers tend to grow a certain species of crop according to market trends, excess production capacity and price collapses are common. Perfect climate conditions can also result in excess production capacity and price collapses. For example, the cold weather in 2014 was suitable for growing garlic, contributing to a 20% growth in production capacity compared with the corresponding period in the previous year. However, the unbalanced production and sales led to a garlic price collapse. Thus, an efficient operating procedure that can obtain accurate information on a large area of agricultural is required. Because of its characteristics of a wide data coverage area, low cost, repeated observation and monitoring, high time efficiency, and the absence of terrain or traffic restrictions, remote-sensing technologies have become crucial survey tools that are gradually replacing conventional time-consuming and laborious ground survey tools. In agricultural management, satellite remote-sensing images have been employed in surveying crop cultivation areas (Gallego, 2004; Pittman et al., 2010; Wan et al., 2010), monitoring crop growth (Doraiswamy et al., 2004; Busetto et al., 2008), predicting crop yield (Doraiswamy et al., 2003; Chang et al., 2005; Wang et al., 2010), identifying spectral difference between tree species (Lawrence et al., 2006; Clark & Roberts, 2012; Dalponte et al., 2012; Naidoo et al., 2012), and analyzing area damage and the extent of loss from natural disasters (Qin et al.,

2008).

Previous studies have adopted various methods and applications with specific advantages and applicability. The aforementioned classification methods are applicable for specific crops and at specific locations and times. However, these methods lack generalizability. However, crops grow continuously with fixed growth cycles, and the spectral reflectance values of crops are affected by the growth conditions (e.g., moisture, nutrition, and disease and pest damages), soil, and differences in natural phenomena such as solar irradiation and atmospheric conditions. These differences can cause slight deviation and irregularity in crop spectral reflectance curves. However, crops grow at a certain level of regularity according to phenological factors. In other words, even though crop growth appears to be irregular, certain patterns of regularity are still observable. Such a situation matches the chaos phenomenon. Thus, in nonlinear systems under certain conditions, the feasibility of applying chaos theory to remote-sensing crop classification is worthy of exploration.

Because spectral reflectance of crops on the ground is affected by numerous environmental factors, it does not present in a regular form. This study applied the chaotic equation to perform a ground-based spectral analysis. Spectral data of different crops were used to model chaotic behavior. The difference among the created chaotic patterns was used to identify and categorize crops. However, chaotic patterns are graph-based representations. They must be quantified to further discern the differences between crops. In a chaotic system, dual attractors form with a specific fractal structure. Therefore, the fractal dimensions of chaotic patterns of different crops can be calculated to identify specific crop categories.

Mandelbrot (1977) indicated that fractals are a phenomenon of complex shapes and irregularity, representing the agglomeration of irregular fragments. Lam and Hsu (2011) suggested that complex natural objects or scenes are formed by fractal patterns, the complexity of which can be effectively quantified through simple iterative methods. Thus, fractal theory can be applied to analyze irregular patterns. Irregular graphs of different crops obtained using the chaotic equation vary in size and shape, and the different fractal structures can be used to categorize the crops on the basis of fractal dimensions.

In this study, garlic, scallion, sweet potato, and carrot were selected as the target crops because they are commonly grown in Yunlin County and were thus convenient for sampling. In conventional image interpretation, scallion and garlic are easily confused, which can lead to misjudging the planting area and yield of both crops. Because these two crops are essential economic crops in Taiwan, distinguishing them is necessary. Sweet potato, and carrot, which are common in the study area and easily confused with garlic and scallion, were also added as targets. Continuous spectral data of different crops were inputted into the model to generate chaotic behavior and chaotic patterns. Subsequently, fractal dimensions were employed to develop a new quantification method for determining whether the fractal dimensions of a certain crop species are within a given range and sufficiently unique to be differentiated from those of other crops. Consequently, the crops can be categorized. MATLAB was used to develop the Chua's circuit chaotic equation before conducting the chaos simulation.

## 2. MATERIALS AND METHOD

### 2.1 Study Area

The sampling location was in the western region of Yunlin County, including Taixi Township and a part of Dongshi Township (Fig. 1). Yunlin County is a crucial crop production region of Taiwan, yielding the highest agricultural production among all counties in Taiwan. Approximately 40% of people living in Yunlin County are farmers. The agricultural land proportion in Yunlin County is also the highest among all counties. The total area of Yunlin County is 129,083 ha, 90% of which is plains and the remaining 10% is mountains. Because Yunlin County is located in a subtropical climate zone, the average annual temperature is 22.6°C and the average annual precipitation is 1,028.9 mm. Taixi Township is located at the west of the Jhuoshuei River alluvial plain. With an area of 66.472 km<sup>2</sup> (approximately 7.2 km east–west and 13.3 km south–north) in a rectangular shape, the Taixi Township is composed of more than 75% plains. Located in the western area of Yunlin County, the sandy loam soil plain of Dongshi Township is 6 km from Taixi Township. The total area of the plain is 48.3562 km<sup>2</sup>.

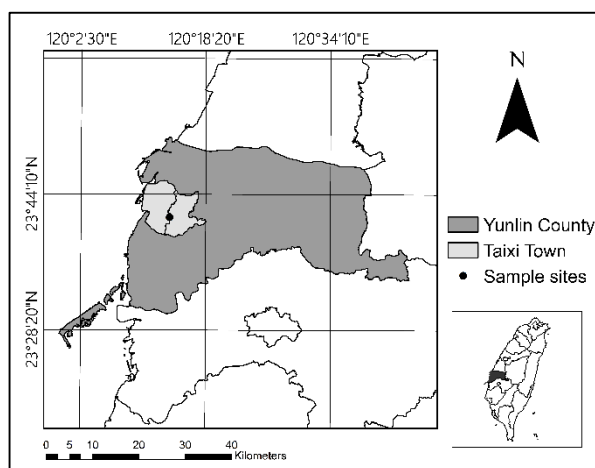


Figure 1. Bitmap of the research area

### 2.2 Collection of Crop's Hyperspectral Reflectance Data

#### 2.2.1 Instruments and procedures of the field work

Crop spectral data (spectral reflectance) in the present study were obtained using PSR-1100 field portable spectroradiometer. The spectral range was 320–1050 nm and the spectral sampling interval was 1.5 nm. The actual number of measured wavebands was 512 and the angle of view was 4°. The properties measured during the field survey included the brightness, luminance reflectance, spectral reflectance, and spectral distribution.

The measurement of reflectance was completed within 2 hours at approximately noon. During sample collection, it was necessary to minimize the interference from manual operation factors. Thus, the measurers wore dark clothes to reduce the errors. When the spectroradiometer was mounted on the tripod, it was placed above the crops and adjusted to avoid the effect of shade from the instrument. Subsequently, the measurer maintained a distance from the spectroradiometer and collected spectral data via a Bluetooth connection between the PDA and spectroradiometer.

### 2.2.2 Field samples collection

From November 2013 to February 2014, the researchers captured the spectral reflectance of samples at the research area on 9 occasions with 7–10-day intervals. The intervals differed because of climate factors, such as samples cannot be measured when it rained. Because the crop planting dates differed according to the climate conditions, a baseline date was selected for subsequent comparisons. Since garlic is the dominant crop in Yunlin County, the date when the garlic was planted in the study plantation (October 1) was used as the baseline date for the analysis. The total sample observations of scallions, garlic, sweet potatoes, and carrots were 81, 81, 27, and 27 for each sampling day.

Before spectral data were collected in the field, a whiteboard was used to simulate the 100% reflectance and prevent changes in the solar incident angles from influencing the results. Thus, after the direct energy (DE) of the plants was captured, the whiteboard correction values recommended by the manufacturer of the spectroradiometer were applied for waveband correction. Finally, the DE values were converted to reflectance values.

### 2.3 Calculation of chaotic equation and fractal dimensions

The objective of the current study was to establish a chaotic pattern that could represent the target crops. The 512 wavebands of one crop at specific time points were substituted into the chaotic equation during each calculation. The Chua circuit chaotic equation developed using MATLAB was used for the simulation. The crop spectral reflectance values were inputted into the equation to generate chaotic patterns of each crop for further calculating its fractal dimensions.

Chua's circuit chaotic equation is a first-order differential equation:

$$\begin{cases} \frac{dI}{dt} = -\frac{1}{L}V_2 \\ \frac{dV_2}{dt} = \frac{1}{C_2}I - \frac{G}{C_2}(V_2 - V_1) \\ \frac{dV_1}{dt} = \frac{G}{C_1}(V_2 - V_1) - \frac{1}{C_1}f(V_1) \end{cases} \quad \text{Equation (1)}$$

Where  $V_1$  and  $V_2$  denote the voltages of the two ends of  $C_1$  and  $C_2$ , respectively, and the current flowing through inductance  $L$  is  $I_3$ . The voltages and current compose a 3D nonlinear element. The 3D trail ( $I(t)$ ,  $V_2(t)$ ,  $V_1(t)$ ) depicts the changes in chaos status. The 2D trail ( $V_1(t)$ ,  $V_2(t)$ ) is called a phase space.

The spectral data were substituted into the Chua's circuit chaotic equation. The 2D phase space ( $V_1(t)$ ,  $V_2(t)$ ) was obtained (Figure 2), in which the X axis represents the voltage at both ends of  $C_1$ , and the Y axis represents the voltage at both ends of  $C_2$ . A chaotic pattern with dual attractors was generated according to the phase space.

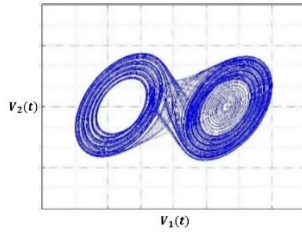


Figure 2. Chaotic pattern involving dual attractors

After the chaotic patterns were created, their fractal dimension patterns were obtained and the differences between them were analyzed inductively and discussed.

Chaotic patterns derived from the chaotic equation reveal different crop types. Fractal dimensions were calculated using Equation (2) to quantify the chaotic patterns and define the fractal dimension range for the chaotic patterns for garlic, scallion, sweet potato, and carrot. The box counting model is expressed in Equation (3):

$$d = \frac{\log N_{\sigma}(F)}{\log \frac{1}{\sigma}} \quad \text{Equation (2)}$$

where  $d$  represents the box count. Figure 3 depicts how the box count is calculated. The box marked as  $F$  in Figure 3 (right panel) shows the minimum box size for the pattern to be measured, which is denoted as  $U$  in the left panel. The smaller box denoted as  $S$  in Figure 3 (right panel) is directly proportional to  $F$ . Let the side length of  $F = 1$  and that of  $S = \sigma$ , the value of which must fall between  $F$  and  $0$ . The side length of  $S$  is one-third the side length of  $F$ .  $N_{\sigma}(F)$  is the  $S$  count needed to cover  $U$  when the side length of  $S = \sigma$ . In Figure 3, the number of  $S$  boxes covering  $U$  is four; thus,  $N_{\sigma}(F) = 4$ . Therefore, calculating the fractal dimensions through the BCM means calculating the logarithm scale, on the basis of which  $N_{\sigma}(F)$  changes with  $\sigma$ . Thus, substituting the parameters in Figure 3 into the equation for calculating fractal dimensions gives  $d = \frac{\log 4}{\log 3} \approx 1.261$ .

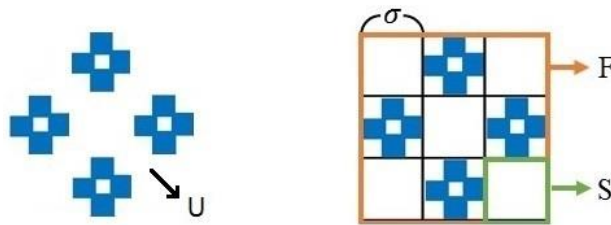


Figure 3. Example of the BCM calculation

The chaotic patterns were also calculated by using the BCM in MATLAB to identify the distribution of fractal dimensions of different crops and determine the sampling time required for distinguishing between them.

### 3. RESULTS AND DISCUSSIONS

This study used MATLAB to calculate the collected spectral ground data on garlic, scallion, sweet potato, and carrot across nine periods. The data were used as voltage strengths in the Chua's chaos circuit equation and analyzed on the basis of identical parameter conditions. The resulting oscillations generate two-dimensional spatial trajectories ( $V_1(t)$ , ( $V_2(t)$ ), which represent a two-dimensional phase space (i.e., the chaotic pattern). The chaotic patterns, as shown in Figures 4–7, reveal differences in their area sizes despite the similarities in the shapes of the patterns. These differences were employed to discern the different crops.

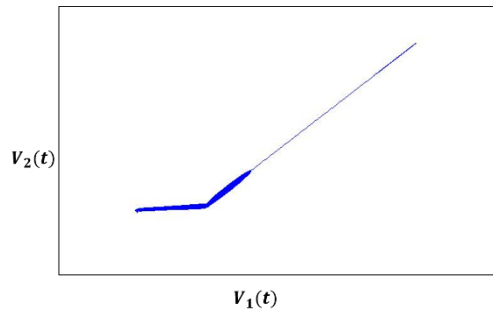


Figure 4 scallions (Jan.16,2014)

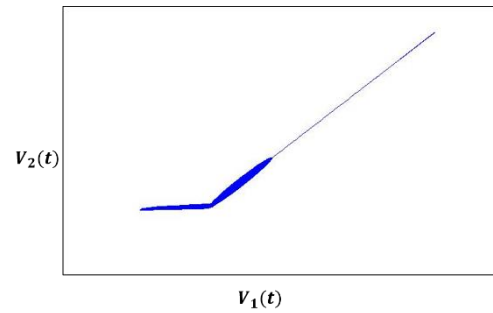


Figure 5 sweet potatoes (Jan.16,2014)

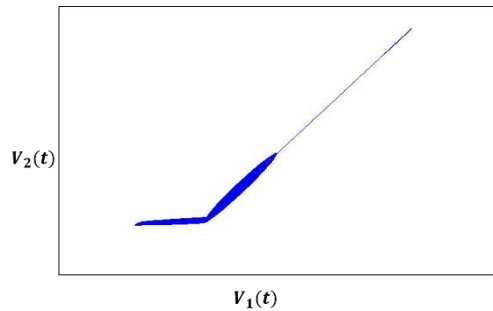


Figure 6 garlic (Jan.16,2014)

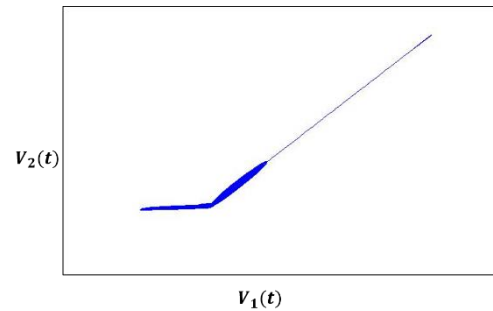


Figure 7 carrots (Jan.16,2014)

After the chaotic patterns were created, MATLAB was used to calculate the fractal dimensions of the crops. To evaluate whether the fractal dimensions could be used to differentiate different crops, the fractal dimension values for each crop category were averaged for each sampling day and the standard deviations (SDs) were calculated because the mean alone would be insufficient for representing the growth of a crop because of the morphological differences between individual crops. After tests were conducted in several ranges, the mean  $\pm$  SD was used as the fractal dimension range for a given crop on a given sampling day (Table 1). Next, whether the dimensions overlapped between crops was determined. Two crop categories presenting nonoverlapped dimension ranges for a given day can thus be differentiated from the fractal dimensions derived from the chaotic patterns.

Table 1 Fractal dimensions of each crop on each sampling day (mean  $\pm$  SD)

|                 |       | 11/12<br>(43)* | 11/26<br>(57) | 12/06<br>(67) | 12/20<br>(81) | 12/24<br>(85) | 01/07<br>(99) | 01/16<br>(108) | 01/28<br>(120) | 02/21<br>(144) |
|-----------------|-------|----------------|---------------|---------------|---------------|---------------|---------------|----------------|----------------|----------------|
| scallion        | +1 SD | 1.313          | 1.291         | 1.308         | 1.290         | 1.295         | 1.295         | 1.295          | 1.312          | 1.330          |
|                 | -1 SD | 1.293          | 1.275         | 1.288         | 1.263         | 1.284         | 1.264         | 1.253          | 1.291          | 1.290          |
| garlic          | +1 SD | 1.306          | 1.316         | 1.329         | 1.304         | 1.302         | 1.304         | 1.317          | 1.317          | 1.315          |
|                 | -1 SD | 1.294          | 1.295         | 1.281         | 1.266         | 1.285         | 1.213         | 1.291          | 1.252          | 1.227          |
| sweet<br>potato | +1 SD | 1.311          | 1.298         | 1.294         | 1.294         | 1.294         | 1.294         | 1.294          | 1.362          | 1.294          |
|                 | -1 SD | 1.294          | 1.294         | 1.282         | 1.280         | 1.266         | 1.285         | 1.271          | 1.294          | 1.293          |
| carrot          | +1 SD | 1.292          | 1.289         | 1.297         | 1.347         | 1.289         | 1.289         | 1.292          | 1.289          | 1.297          |
|                 | -1 SD | 1.289          | 1.273         | 1.289         | 1.289         | 1.256         | 1.267         | 1.289          | 1.277          | 1.289          |

\* The values in parentheses represent the number of days since October 1, which was the baseline date when the garlic crops were planted.

The results revealed that on November 12 (Day 43), the fractal dimension ranges for the carrot crops (dimension range: 1.289–1.292) were unique from the other three crops (Figure 8). On November 26 (Day 57), the scallion (dimension range: 1.275–1.291) and garlic crops (dimension range: 1.295–1.316) were unique from each other (Figure 9); however, the carrot crops were confused with the scallion crops, and the sweet potato crops were confused with the garlic crops.

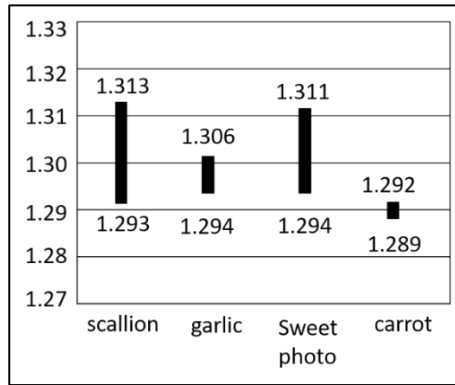


Figure 8. Crop's dimensions add and subtract one standard deviation on Nov. 12, 2013.

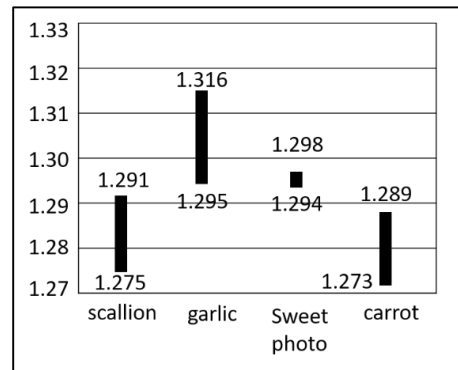


Figure 9. Crop's dimensions add and subtract one standard deviation on Nov. 26, 2013.

The fractal dimension ranges for the carrot crops (dimension range: 1.289–1.292) were unique from the other three crops.

The scallion (dimension range: 1.275–1.291) and garlic crops (dimension range: 1.295–1.316) were unique from each other

## 4. CONCLUSIONS AND SUGGESTIONS

### 4.1 Conclusions

A chaotic equation was used in a ground-based hyperspectral analysis. Spectral data of the crops were input into a model to generate chaotic behavior. Through nonlinear concepts, various crop models were established. According to previous studies, hyperspectral crop data substituted into a chaotic equation can generate chaotic behavior because crop spectra possess consistently high sensitivity to slight differences and features continuity. Thus, chaos theory can be applied to hyperspectral image-based crop classification.

This study compared the quantified fractal dimensions of chaotic patterns of crop spectral reflectance values calculated individually for nine sampling days. Using the quantified dimensions for each sampling day, we evaluated the appropriate time for crop differentiation. The results may serve as a reference regarding which dates crop spectral reflectance data should be collected for crop differentiation.

When the fractal dimension ranges of crops of a given day are nonoverlapping, the crops can be differentiated. An example of this is the differentiation of the carrot crops on November 12, which were



unique from the other three crops. In image interpretation, confusion regarding which crops are which occurs frequently. For example, the spectral reflectance of garlic is similar to that of scallion, and this could easily cause confusion. When the chaotic pattern and fractal dimension calculation method proposed in this study was used, scallion and garlic were distinguishable from each other on November 26 if the data for sweet potato and carrot were not considered.

#### **4.2 Suggestions**

Because chaos theory was applied in remote sensing for the first time, much improvement can be achieved in the future. According to the problems encountered in the research process, the following suggestions are provided for future studies.

In our research outcomes, there was still some confusion in distinguishing between crops. Suggestions for crop data collection are thus proposed as follows. First, samples of the same species of crops can be collected from multiple locations. A portion of the samples can be used to establish ranges that represent specific crops (e.g., mean  $\pm$  SD), whereas the other samples can be used to verify the accuracy of the established ranges. Second, identical sample sizes must be used for different crop categories to ensure that the data used in the analysis are consistent between crops. In addition, complete phenological data on crops in different growth phases must be collected to create a comprehensive spectral database conducive to crops differentiation.

This study used the chaotic equation to derive chaotic patterns, which were quantified using the fractal dimension of the BCM. Subsequently, dimension ranges were employed to discern between different crops. Future research should consider exploring whether using the maximum/minimum value derived from the chaotic patterns can facilitate crop categorization, or where other feasible quantification methods can be used to more accurately define the chaotic patterns derived from the model.

#### **5. References**

- Busetto, L., M. Meroni, and R. Colombo., 2008. Combining medium and coarse spatial resolution satellite data to improve the estimation of sub-pixel NDVI time series. *Remote Sensing of Environment*, 112: 118-131.
- Chang, K.W., Y. Shen, and J.C. Lo., 2005. Predicting rice yield using canopy reflectance measured at booting stage. *Agronomy Journal*, 97:872-878.
- Clark, M. L. and D.A. Roberts, 2012. Species-level differences in hyperspectral metrics among tropical rainforest trees as determined by a tree-based classifier. *Remote sensing*. 4:1820–1855.
- Dalponte, M., L. Bruzzone and D. Gianelle, 2012. Tree species classification in the Southern Alps based on the fusion of very high geometrical resolution multispectral/hyperspectral images and LiDAR data. *Remote Sensing of Environment*. 123:258–270.
- Doraiswamy, P.C., J.L. Hatfield, T.J. Jackson, B. Akhmedov, J. Prueger and A. Stern. 2004. Crop

condition and yield simulations using Landsat and Modis. *Remote Sensing of Environment*, 92: 548-559.

Doraiswamy, P.C., S. Moulin, P.W. Cook and A. Stern., 2003. Crop yield assessment from remote sensing. *Photogramm. Eng. Remote sensing*, 69: 665-674.

Feigenbaum, M. J., 1983. Universal behavior in nonlinear systems. *Physica*, 7: 16-39.

Gallego, F. J. 2004. Remote sensing and land cover area estimation. *International Journal of Remote Sensing*, 25: 3019-3047.

Guess, D. and W. Sailor., 1993. Chaos theory and the study of human behavior: Implications for special education and developmental disabilities. *Journal of Special Education*, 27(1):19-34.

Kauffman, S. A., 1991. Antichaos and adaptation. *Scientific American*, 265: 78-84.

Kelsey, D., 1988. The economics of chaos of the chaos of economics. *OxfordEconomic Papers*, 40: 1-31.

Lam, A.D.K.T., Hsu, P.Y. 2011. Fractal analysis for textile pattern design. *J. Cult. Creat. Ind. Res.* 4 (1), 221-235.

Lawrence, R.L., S.D. Wood and R.L. Sheley., 2006. Mapping invasive plants using hyperspectral imagery and Breiman Cutler classifications (randomForest). *Remote Sensing of Environment*. 100:356–362.

Lorenz, E. N., 1963. Deterministic Nonperiodic flow. *Journal of the Atmospheric Sciences*, 20(2):130-141.

Mandelbrot, B.B., 1977. *Fractals: Form, Chance and Dimension*, W.H. Freeman and Company, San-Francisco.

Naidoo, L., M.A. Cho, R. Mathieu and G. Asner, 2012. Classification of savanna tree species, in the Greater Kruger National Park region, by integrating hyperspectral and LiDAR data in a Random Forest data mining environment. *ISPRS Journal of Photogrammetry and Remote Sensing*. 69:167–179.

Pittman, K., M.C. Hansen, I. Becker-Reshef, P.V. Potapov, C.O. Justice, 2010. Estimating global cropland extent with multi-year MODIS data. *Remote sensing*, 2: 1844-1863.

Qin, Q., A. Ghulam, L. Zhu, L. Wang, J. Li and P. Nan. 2008. Evaluation of MODIS derived perpendicular drought index for estimation of surface dryness over northwestern China. *International Journal of Remote Sensing*, 29: 1983-1995.

Strunk, V. L., 1995. A study of transition boundaries in organizational evolution: The punctuated equilibrium model. Walden University, Unpublished Dissertation. AAC95336779.

Trygestad, J., 1997. Chaos in the classroom: An Application of chaos theory. Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL. ED413289.

Wan, S., T.C. Lei and T.Y. Chou., 2010. An enhanced supervised spatial decision support system of image classification: consideration on the ancillary information of paddy rice area. *International Journal of Geographical Information Science*, 24: 623-642.

Wang, Y.P., K.W. Chang, R.K. Chen, J.C. Lo and Y. Shen, 2010. Large area rice yield forecasting using satellite imageries. *International Journal of Applied Earth Observation and Geoinformation*, 12: 27-35.