Accurate interpolation of Spectral Response Functions of Ocean Colour Monitor

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Abstract: Precise mathematical representation of spectral response functions (SRF) of Earth Observation sensors plays a key role in calibration exercises and also for out-of-band signal compensation in the operational phase of these sensors. With narrow spectral bandwidths and high radiometric resolutions built with ocean colour sensors (Moderate resolution imaging spectrometer-MODIS, Medium resolution imaging spectrometer-MERIS, Sea viewing wide field of view sensor-SeaWIFS, Ocean colour monitor-OCM) in order to extract small signal from the ocean upwelling radiance in the high background atmospheric scattering, more accurate representation of the SRF assumes significant importance. The SRF measurements are typically carried out at about 1nm intervals in the in-band spectral region and at about 10nm intervals in out of band regions. In this paper we investigated a detailed interpolation analysis of the SRF of Ocean Colour Monitor flown on-board Indian Oceansat-2 mission. Several methods have been reported in the literature. Of these, it is found that the OCM SRF data is best fitted with B-spline interpolation method, and hence was adopted in the present study. Criteria for resampling rate was worked out and spectral interpolation covering all the spectral regions was carried out. The interpolated data was analyzed in detail for band specific parameters (peak wavelength, centre wavelength and full width half maximum). We found that cubic B-spline based interpolation function is the optimal method for representing the SRF covering all the spectral regions in terms of arriving at a smooth function to compensate noise in the laboratory recording process, and also providing accurate interpolated values in the gaps between samples.

Key words: Ocean Colour Monitor, relative spectral response, cost function, peak error, smoothness control.

1. Introduction

Today, space based ocean colour remote sensing is being carried out operationally worldwide. It started with the launch of Coastal zone colour scanner (CZCS) in 1978 and it has progressed into operational phase with the launch of Sea viewing Wide field sensor (SeaWiFS) in 1997 (IOCCG, 2015). Subsequently, many Ocean colour sensors (OCS) like Ocean Colour Monitor (OCM, Indian Space Research organization, ISRO, 1999 and 2009) (IOCCG, 2016) and MODIS-Acqua and Terra (NASA, 1999, 2002) were launched and are providing high quality ocean colour data. OCS measure top of the atmosphere (TOA) radiance with very low noise 10 bit systems (0.1% accuracy). TOA radiance is dominated by the contribution of atmospheric scattered light (about 90%). And the upwelling radiance from the ocean, which carries the information on water contents, constitutes about 10% of signal. Atmospheric scattered as well as the upwelling radiance vary significantly with wavelength depending on the constituents. OCS acquire image data, typically, in 8 to 10 spectral channels in visible and near Infrared (VNIR) region of electro-magnetic spectrum. This data is being used for retrieval of information on ocean water contents like Chlorophyll, sediments etc. Retrieval process involves complex algorithms tuned for each sensor, location, season and application involving (a) radiometric and geometric correction of data (b) computation of TOA spectral radiance (c) estimation of spectra of surface reflections, atmospheric scattering etc, (d) measurement of in-situ upwelling radiance at sample points and (e) modelling (Mobley,2011 and Kwaitkowska, 2006). Spectral response function (SRF) of OCS plays an important role in accurate retrieval of ocean upwelling radiance. SRF is the absolute response of the sensor with respect to wavelength and denotes the spectral transmittance of the sensor. It is generally normalized with respect to peak value and referred as relative spectral response (RSR). The peak values of SRF are used in the radiometric part of system transfer function. SRF is measured in the laboratory at discrete wavelengths covering visible and near IR range. The spectral bandwidth (FWHM-Full width half maximum) of these sensors is about 10-20nm. RSR data is available from space agencies at varying intervals. For example, original RSR measurements of OCM are available at 1/2nm intervals in and around FWHM and at about 4/6/10nm intervals in the rest of VNIR spectral range. This RSR needs to be defined as a precise mathematical function and RSR values need to be arrived at accurately at higher sampling intervals for carrying out many scientific studies including computation of spectral radiance (TOA).

Convolution of RSR is carried out with computed spectra of atmospheric scattered light etc, to arrive at their contribution to the signal. Goddard space flight center's Ocean biology group provides interpolated RSR of SeaWiFS at 1nm intervals while the RSR of MERIS is available at 0.1nm.intervals. MODIS RSR data is available at 1nm interval after interpolation, though the merged (in-band and out of band) data is available at non-uniform samples. Some of the future OCS systems will have higher bit digitization and narrower bandwidths.

Considering the requirements of accurate interpolation, we carried out survey of the literature and identified suitable approach for interpolation and resampled data to very high resolution. B-spline based approach was developed for RSR interpolation. This method was validated using RSRs of MERIS and other OCS data, which simulate varying shapes (slopes) and sampling intervals. Subsequently, it was applied on RSRs of OCM. We also assessed the accuracy of interpolated RSR of OCM compared to original data. In this paper we present the details of accurate interpolation approach followed and the results of the interpolated data. Section-2 gives a summary of interpolation methods. Section-3 provides details of materials and methodology used. Section-4 discusses results and summarizes the study.

2. Interpolation approach

Interpolation is a problem of constructing continuously defined mathematical function using the given discrete data and then arriving at values at intermediate locations. It has been a very interesting subject since ancient astronomy days. Large number of techniques were conceived by ancient Babylonian as well as medieval astronomers and mathematicians from China, India and Arabia. For example, Indian astronomer-mathematician Brahmagupta (625AD) proposed a method for second order interpolation of sine and versed sine functions (Erik Meijering, 2001). Subsequently, many classical techniques were developed by western countries in 17^{th} to 19^{th} centuries. Among them, Newton gave very important formulae using divided differences. The problem of interpolation was further studied by many researchers including Stirling, Gauss, Waring, Euler, Lagrange, Bessel, Laplace, Everett, Jenkins, Taylor, Runge, Borel, Whittaker, Shanon, Schoenberg and others. Subsequent to development of approximation theory, the limitations of algebraic formula based interpolation were realized. Subsequently, researchers approached interpolation using cardinal functions, osculatory functions and convolution. Cubic convolution (Sinc like kernel) has been extensively used in digital image processing since 1970. Shanon (1949) showed that a band limited function can be reconstructed from its equidistant samples using pulses (Sinc function: Sinπx/πx type as the basis function). Schoenberg (1946) noted that the basis function Sinc is inadequate for numerical purposes as it has excessively low damping rates. He showed that B-Spline (Basis spline) based approach does not lead to convergence difficulties as it vanishes after few knots (data points). B-splines are polynomials (for example B-splines of degree 3 comprise cubic polynomials) joined at a set of knots (boundaries). This structure allows for controlling smoothness at various knots along the curve. Each strand can be parameterized in an optimal fashion. Sergey Fomel (2001) carried out detailed evaluation of techniques of nearest neighbor interpolant, linear interpolant, cubic convolution interpolant, Sinc interpolant, B-spline, B-spline interpolant (cardinal B splines) by interpolating 50 point uniformly sampled one dimensional signal (with characteristics of exponential amplitude decay and quadratic frequency decrease) to 500 point samples. He discussed the frequency response of various interpolants. Sinc function based interpolation has flat unity relative frequency response upto Nyquist and its response is zero beyond Nyquist. Sinc like interpolations have smooth roll off frequency responses extending beyond Nyquist. He also showed that B-spline based approach provides more accurate and smooth response with frequency response extending beyond Nyquist. Similarly, Lehmann (1999) recommended B-spline based approach for interpolation applications in medical image processing after detailed assessment of various methods. Fourier series can be used as basis function for periodic curves. When the curves are not periodic and are complex, most commonly used basis function is B-splines (Levintin, 2007, Mathworks, 2017) due to their compact support requirements. Some iterative methods were reported using Sinc function etc. for interpolating non-uniformly sampled data and these methods require long computational support. From the above studies, it came out that B-Spline based approaches are the best with regard to minimizing errors as they combine a maximal order of approximation with maximum regularity. Also, RSR is expected to be smooth function as spectral filters are made of large number of layers of interference coatings and the input light is a convergent beam in our case. It may be noted that very few papers, like the ones mentioned below, have discussed on interpolation of RSR. There were no reports on detailed evaluation of the accuracy interpolated RSR of OCS. Cubic spline approach was reported to improve spectral mis-registration in imaging spectrometer (Yutao Feng, 2008). It is also important to impose conditions of maximizing correlation between original dataset and the interpolated data set (James L Gardner, 2003).

3. Methodology and Materials

We obtained RSR data of OCM from ISRO and that of MERIS and other sensors from GSFC website which were available for research. Original RSR measurements of OCM were available at 1/2nm interval in FWHM region and at 4/6/10nm intervals in the rest of VNIR spectrum of 400 to 1100nm (about 112 points). RSR measurements of

MERIS were available at 0.1nm intervals and those of SeaWiFS and MODIS were available at 1nm intervals for complete VNIR spectral range. It may be noted that RSR data from GSFC is available after interpolation, though the details of the method followed is not available. Table-1 shows the sampling characteristics of RSR data of various OCS.

OCS are generally precision systems compared to terrain imaging sensors with 10 bit digitization and high dynamic range. In our research on OCS, computations involving TOA spectral radiance and RSR are planned to be carried out commensurate with 10 bit accuracy and accordingly LSB (least significant bit) corresponds to 0.1% of full scale. As FWHM is about 20nm, bandwidth contributing to 1 LSB equivalent signal would be about 0.02nm. Hence, the resampling of RSR, after interpolation is planned to be carried out at 0.02nm sample intervals (about 37500 sample points). Though the impact of sampling interval outside FWHM will be less than that of the same within FWHM, the sampling interval was retained as 0.02nm throughout the VNIR spectrum.

Based on the literature study discussed in section 2 on the interpolation methods, we selected B-spline method for interpolation of RSR data of OCS including OCM (non-uniform sampling). By this selection, we imposed conditions that RSR and its derivatives are smooth with no discontinuity. Third degree B-splines are considered here as they provide continuity upto second derivative and also higher degree B-splines may lead to more smoothening there by attenuating high frequencies. B-Spline are centered around each data point (say knot K) with end points becoming zero after two knots on either side (K±2). Additionally, knots are generated on either side of data set to provide the continuity of the data at end points. Interpolated function, mathematical representation of RSR data is shown below as a linear combination of B-splines.

$$fx_i(\Delta'\lambda) = \sum_{k=1}^{K} c_{ik}\varphi_k(\Delta'\lambda) ------ (1)$$

Where

$fx_i(\Delta'\lambda)$ = Value of Interpolated RSR function at i^{th} Interpolated interval

$\Delta'\lambda$ = Spectral Sampling interval of the interpolated RSR data

K = Number of B-Splines = No of samples in the original data +2

$\varphi_k(\Delta'\lambda)$ = Value k^{th} B-Spline of 3^{rd} degree defined at $\Delta'\lambda$ intervals

C_{ik} = values of set of basis coefficients

The coefficients c_{ik} are optimized by minimizing cost function, *F(c)* as below

$$F(\boldsymbol{c}) = \sum_{j}^{n}\left[fx_j(\Delta\lambda) - \sum_{k}^{K} c_{jk}\,\varphi_k(\Delta'\lambda)\right]^2 + \beta\int[D^2 fx_j(\Delta'\lambda)]^2 d\lambda ------(2)$$

Where $fx_j(\Delta\lambda)$ = Original RSR data at j^{th} Knot

$\Delta\lambda$ = Spectral Sampling interval in the original RSR data

$n =$ Number of samples in the original RSR data/ Knots

$\beta\int[D^2 fx_j(\Delta'\lambda)]^2 d\lambda$= Total curvature of interpolated RSR function (Roughness penalty)

$D^2 fx_j(\Delta'\lambda)$ = Second derivative of interpolated RSR function at j^{th} knot

β = curve smoothness control parameter

As can be seen from equation-2 (second term), weightage is given to curvature (smoothness) in addition to the correlation of the original and interpolated data (first term). The coefficients C_{jk} are obtained by regression for each value of β. Figure-1 shows typical B-splines for one location in the RSR data set of OCM-2. The curve control parameter, β is varied from 10^{-8} to 10 in steps of $10^{-0.0025}$. This range and fine steps are incorporated to characterize the impact of smoothness control over interpolation as well as to achieve high accuracy. Higher values of β indicate that the curve is made smoother. Final set of C_{jk} values is selected corresponding to minimum value of cost function Fc_{min} and interpolated values of the RSR function worked out at 0.02nm intervals. The code for computations are developed in R-software using its libraries and implemented in desk top computer.

RSR interpolation was carried out for all channels of OCM-2, SeaWiFS, OCM-1, MODIS-A, MODIS-T, VIIRS and MERIS as they simulate various shapes of RSR function and also various sampling rates. Table-2 shows the complexity of RSR data studied. It can be seen from the table that the maximum gradients varied from 0.1 per nm (slow variation) to 0.7 per nm (steep variation), 1 being the maximum value of RSR. To further assess the robustness of cubic B-spline approach for RSR interpolation, we decimated 10% of the samples randomly in the original OCM-2 data, there by simulating irregular samples (in addition to the fact OCM-2 has non-uniform sampling) and interpolation was carried out. The irregular sampling also simulates the effect of errors in the experiment, if any, in wavelength measurements.

The accuracy of interpolation was assessed by comparing parameters of RSR, namely Peak wavelength (λp), Centre wavelength (λc) and Full width half maximum (FWHM) computed before and after interpolation. The values of cost function Fc, which indicates the goodness of fitness, were analyzed. The deviation of interpolated data compared to original was analyzed at all input points. The above analysis method was developed using R-software and Excel.

4. Results and discussion

RSRs were interpolated for all Ocean colour channels of OCM-1, OCM-2, SeaWiFS, MODIS-A, MODIS-T, MERIS and VIIRS with the above procedure. Figure-2 shows a comparison of original and interpolated data for some typical channels highlighting the in-band area. Interpolated RSRs match well with the originals except for few locations. Data and plots show that the interpolation method worked very well in generating spectral data at 0.02nm intervals for all RSRs of various shapes. This method brought out the mathematical relationship in the data to carryout interpolation and smoothened the random errors. It also minimized the sampling errors in the original data. It can be seen that all transitions are matching very well. As can be seen from the graphs of OCM-2 Band-1 (around 430-440nm) and OCM-2 Band-2 (around 410 to 420nm), the errors in the original data due to change of sampling rate in the original data (from 1nm interval to 10nm interval) was estimated by the B-spline based interpolator using functional relationship of adjacent data and interpolated properly. Similarly, in case of MODIS-A Band 8 (around 410nm), the lack of information (in the original data) due to sampling was interpolated using the data variations on both sides. Smoothening of few points in the pass band was found though the transitions and systematic variations were retained. This smoothening is expected as 3^{rd} degree B-spline is a low pass filter.

The accuracy of interpolated RSR of OCS was checked by evaluating RSR parameters. Tables 3A and 3B show comparison of RSR parameters of OCM-2 and SeaWiFS respectively. Peak wavelength, Central wavelength and FWHM match very well and these values are available up to 2^{nd} decimal accuracy after interpolation. The difference between the original and interpolated RSR parameters has been less than 1nm commensurate with sampling limitation of original data. This has been observed in all the channels and sensors studied. The cubic B-Spline based interpolation retained spectral characteristics and improves the accuracy.

The accuracy of interpolation was also checked as a data set. Table-4 shows goodness of fitness for various sensors and channels. It is observed that the values of cost function Fc_{min} are very small and are in the range of 10^{-18} to 10^{-3} even though each value consists of fit errors (sum of squared deviations) of about 110 points and curvature of the complete curve. The RSS error works out to be less than 0.00264. It shows that excellent correlation exists between original data and the data generated with cubic B-spline based interpolation. The smoothness control values corresponding to F_{cmin} are in the range of 10^{-8} to 0.178. Figure-3 shows typical variation of cost function Fc with respect to values of smoothness control parameter β. It is also observed from the plots that Fc increases significantly after we give more emphasis on smoothening (β > 0.1/0.2) for regression in all the cases. Very small values of β indicate that B-spline based function represents a smooth curve connecting all the points and fits very well with the functional relationship among the data points. Very small values of Fc and very small values of β corresponding to Fc_{Min} in all the channels also indicate that original data was having less errors/noise except in few points of few channels.

The deviations between the original and interpolated RSR values are studied in detail. Table -5 shows worst case deviation among all the input points of 8 channels of OCM-2. Though the deviations are very small, channels 1 to 3 of OCM-2 have shown higher deviations than the other channels. A maximum deviation of 0.0162 was observed for Channel-1 but at very few points. Figure-4 shows spot deviations of data points in the in-band region and also in overall VNIR range of RSR for typical channels. The number of such peak deviations are very few. As expected, in most of the cases, the deviations are present in the in-band region, where the data variations are more. Though the peak deviation values are very less, deviation pattern follows RSR profile in two bands. Fc_{Min} values are slightly higher for those bands with higher peak deviations compared to others. These deviations in OCM-2 could be due to noise in measurements. Effect of irregular sampling on interpolation was studied by decimating 10% of measurements randomly in OCM-2 and typical results are summarized in Table-6. It shows the wavelengths at which decimation was introduced, the RSR values in the original measurement, the RSR values after interpolation without any decimation and the RSR values after decimation and interpolation. It can be seen that the decimation was simulated even for consecutive points. The differences between the values of RSR with decimation and interpolation compared to original are very less and are within the noise range. Considering analysis of all the deviations and the values of cost function, it can be said that cubic B-spline based interpolation gave very accurate mathematical representation of the original RSR data sets.

5. Summary and Conclusions:

Accurate interpolation of RSR is required for ocean colour sensors commensurate with 10 bit digitization. Large amount of research was carried out on interpolation approaches since ancient times. For non-uniformly sampled data, like in OCM-2, B-spline based interpolation was best choice. The RSR interpolation method was developed using cubic B-splines in R-Software. Large number of RSRs simulating variations in shapes and sampling were interpolated for various values of smoothness control parameter and the one with least value of cost function was selected. The accuracy of interpolated data was assessed in terms of spectral parameters, cost function values (sum of squared differences and curvature) and deviations at individual points. The robustness of B-spline approach was studied for irregularly sampled data with respect to accuracy.

Based on the detailed study it can be concluded that the cubic B-Spline based interpolation brings out the mathematical relation among the data points, provides accurate interpolation of RSR of Ocean Colour Monitor and other OCS and helps to generate data at very high sampling intervals.

Acknowledgements: We are grateful to Indian Institute of Remote sensing (IIRS), Space Applications Centre (SAC) and National Remote Sensing Centre (NRSC) of ISRO and Doon University (DU) for supporting this research work. Our sincere thanks to NASA, GSFC for the RSR data. We gratefully acknowledge the useful discussions we had with Dr Prakash Chauhan, SAC, Dr Sameer Saran, IIRS, Dr Sarnam Sigh, IIRS and Dr Vijay Sridhar, DU.

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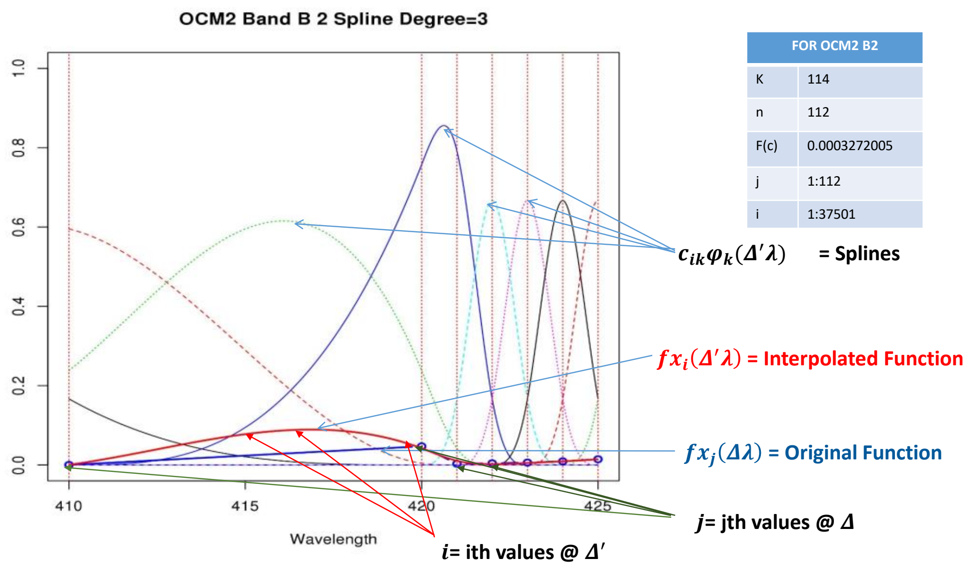
Fig-1: Snapshot of B-Splines at around 420nm for Fc Min condition



FOR OCM2 B2	
K	114
n	112
F(c)	0.0003272005
j	1:112
i	1:37501

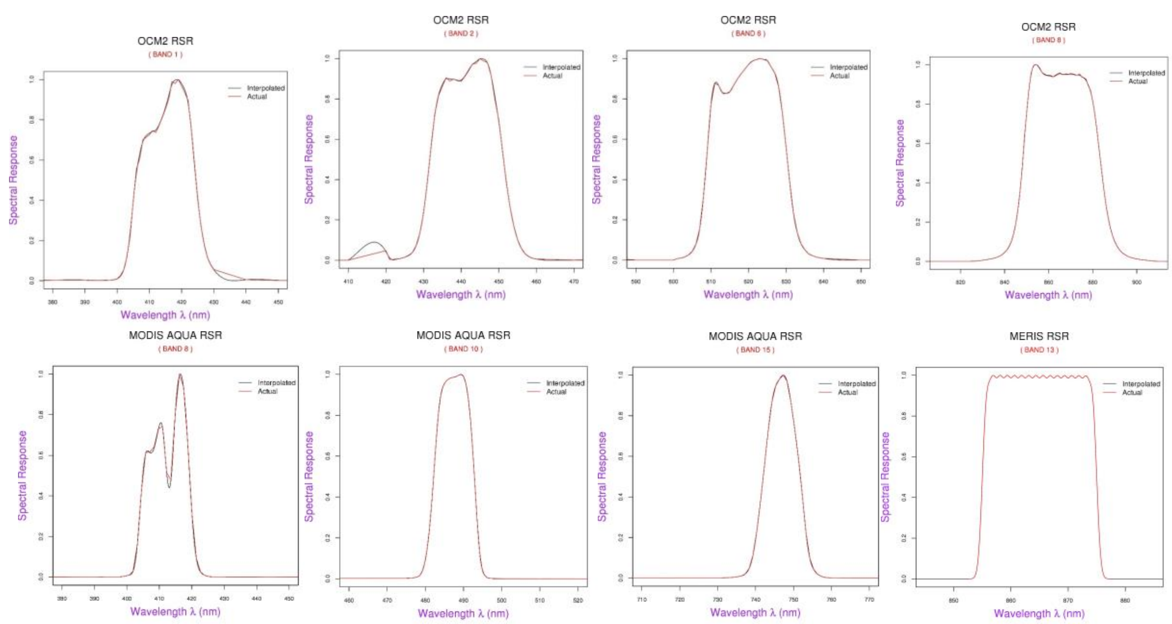


Fig- 2: Comparison of original and interpolated data

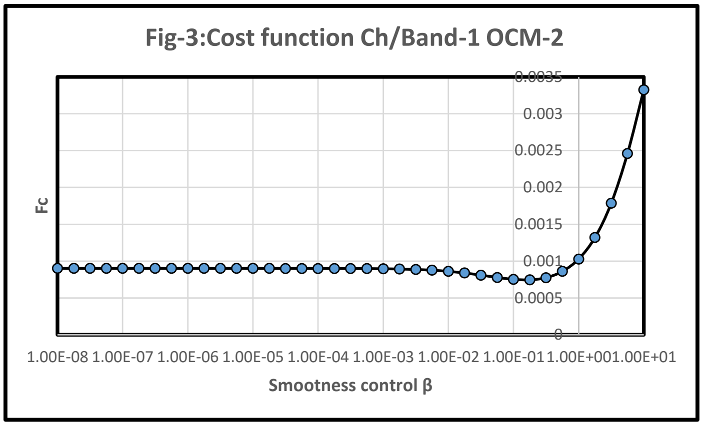


Fig-3:Cost function Ch/Band-1 OCM-2

Fig-4: Spot Deviations between Original and Interpolated (Typical results)

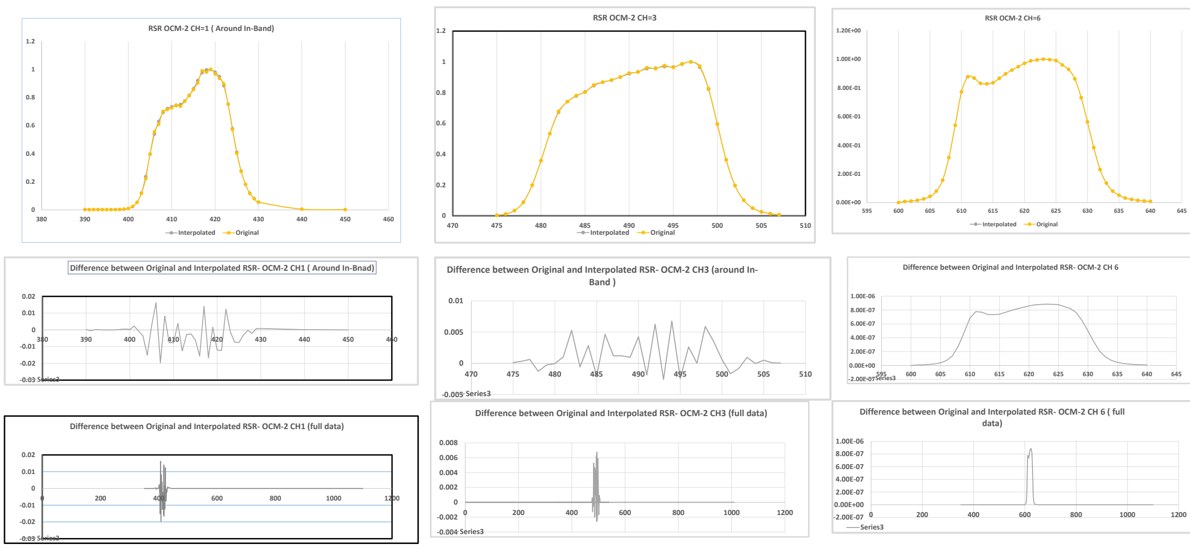


Table-1: Characteristics of Original RSR data of various OCS- A summary (values in nm)

Sensor	No of channels	In-band interval	Out of band interval	Starts at	Source
OCM-1	**8**	**1/2**	**4/6/10**	**346**	**ISRO**
OCM-2	**8**	**1**	**10**	**350**	
SeaWiFS	**8**	**1**	**1**	**380**	**NASA Goddard Space flight Centre**
MODIS-A/T	**9**	**1**	**1**	**380**	**http://oceancolor.gsfc.nasa.gov/DOCS/RSRtables.html (updated 13 July 2012)**
MERIS	**15**	**0.1**	**0.1**	**405**	

Table- 2: Complexity of RSR data for various OCS (Max Gradient/nm)

Channel	OCM-2	VIIRS	MODIS-A	MODIS-T	MERIS
1	-0.18	-0.33			0.70
2	0.18	0.26			0.68
3	-0.24	0.35			0.67
4	-0.23	0.23			0.66
5	0.21	0.16			0.66
6	0.25	0.24			0.67
7	0.20	0.13			0.68
8	0.14		0.28	0.31	0.68
9			-0.29	-0.28	0.69
10			-0.30	-0.30	0.70
11			0.24	0.23	0.70
12			0.22	0.22	0.70
13			0.20	0.20	0.71
14			0.17	0.17	0.72
15			-0.20	-0.19	0.72
16			-0.15	-0.15	

Table- 3A: Comparison of RSR parameters (nm) before and after Interpolation – OCM-2

Channel	Peak wavelength		Centre wave length		FWHM	
	Interpolated	Original	Interpolated	Original	Interpolated	Original
1	418.62	419	415.07	415	18.74	18
2	445.38	445	441.46	441.5	19.6	19
3	497.02	497	490.6	490.5	19.56	19
4	517.54	517	511.95	511.5	18.98	19
5	560.94	561	556.75	557	17.86	18
6	623.02	623	619.6	619	21.48	21
7	734.4	734	743.98	744	30.6	30
8	854	854	865.88	866	35.44	36

Table- 3B: Comparison of RSR parameters (nm) before and after Interpolation – SeaWiFS

Channel	Peak wavelength		Centre wave length		FWHM	
	Interpolated	Original	Interpolated	Original	Interpolated	Original
1	417.92	418	413.29	413	20.1	20
2	446.18	446	443.94	444	19.6	20
3	487.26	487	491.12	491	20.52	20
4	514.6	514	510.07	510	22.34	22
5	557.64	558	554.63	554.5	18.26	19
6	670.82	671	668.23	668	19.82	20
7	774.58	775	764.87	765	40.22	40
8	858.68	858	866.35	866.5	41.34	41

Table-4: Goodness of Fitness for various channels and sensors

Channel	OCM-2		VIIRS		MODIS-A		MERIS	
	Fc_{MIN}	β	Fc_{MIN}	B	Fc_{MIN}	β	Fc_{MIN}	β
1	0.000746	0.178	1.61E-06	1.00E-08			1.41E-11	1.00E-08
2	0.000327	0.1	9.68E-07	1.00E-08			8.91E-12	1.00E-08
3	0.000166	0.032	1.53E-06	1.00E-08			6.69E-12	1.00E-08
4	2.60E-05	0.01	2.27E-06	1.00E-08			1.02E-11	1.00E-08
5	2.71E-05	1.00E-08	8.13E-07	1.00E-08			9.71E-12	1.00E-08
6	8.42E-05	1.00E-08	8.93E-08	1.00E-08			9.63E-12	1.00E-08
7	3.26E-05	0.0178	4.88E-08	0.01			1.45E-11	1.00E-08
8	2.72E-05	0.0178			0.000588	1.0E-08	1.13E-11	1.00E-08
9					6.44E-05	1.0E-08	1.25E-11	1.00E-08
10					3.03E-06	1.0E-08	1.17E-11	1.00E-08
11					7.57E-09	1.0E-08	9.29E-12	1.00E-08
12					5.79E-07	0.000178	1.64E-11	1.00E-08
13					8.77E-05	1.0E-08	2.38E-11	1.00E-08
14					2.06E-06	1.0E-08	1.74E-11	1.00E-08
15					4.43E-06	1.0E-08	1.36E-10	1.00E-08
16					3.44E-09	1.0E-08		

Table-5: Original Vs Interpolated data (OCM-2)

Channel	No of data points	Peak deviation
1	112	0.016245759
2	112	0.006495055
3	112	0.006766
4	112	0.004842778
5	112	0.0000744
6	112	0.000000886
7	112	0.003461302
8	133	0.009031

Table 6: Effect of irregular sampling on B-spline based RSR Interpolation

B1				B5			
λ_D	RSR_O	RSR_{OI}	RSR_{DI}	λ_D	RSR_O	RSR_{OI}	RSR_{DI}
391	0.001378	0.001851	0.002754	539	0.0005	0.000465	0.000335
392	0.00175	0.001599	0.002284	559	0.9705	0.970427	0.970462
393	0.001653	0.001609	0.001847	571	0.0263	0.026271	0.026316
395	0.001529	0.001543	0.001463	577	0.0004	0.000378	0.000391
416	0.903792	0.919234	0.917412	620	0.0000	4.80E-05	2.55E-05
418	0.980157	0.996935	0.9998	650	0.0000	2.10E-05	0
470	0.000431	0.000434	0.000917	690	0.0001	0.000116	0
560	0.000259	0.00026	0.000278	760	0.0001	0.000132	6.19E-05
620	0.000262	0.000264	0.000305	770	0.0000	2.90E-05	4.33E-05
650	0.00027	0.000271	0.000255	790	0.0000	2.30E-05	2.02E-05
770	0.000315	0.000317	0.000316	910	0.0000	2.30E-05	2.27E-05

Notes: Decimation carried out randomly at 10% points of original RSR,
Notations λ_D : Wavelength Deimated, RSR_O = Original RSR,
RSR_{OI} = RSR After interpolation of Original data, RSR_{DI} = RSR After Decimation & interpolation